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ON THE BEHAVIOR OF STEAM IN THE STEAM ENGINE CYLINDER, AND ON CURVES OF EFFICIENCY.

A paper read before the New York Academy of Sciences, Feb. 13th, 1882.

By ROBERT H. THURSTON, Fellow of the Academy.

In an earlier paper* the writer endeavored to show what are the conditions determining the ratio of expansion in steam engines working at maximum efficiency, and how those conditions vary in different types of engine, and also how essentially different is the actual working engine, in all that affects that ratio, from the hypothetical case usually taken, in which the steam is assumed to expand adiabatically in a non-conducting steam cylinder. It was finally shown what were the best values of this ratio for several standard types and representative cases, as determined by the writer by direct observation and by the study of experimentally obtained results, the precise figures given being obtained by rules of simple form so deduced.

It was shown that first friction and then—often to a vastly greater extent—cylinder condensation, due to expansion of a heated fluid in a working cylinder made of a material of high conducting power, mod-

* On the Ratio of Expansion at Maximum Efficiency in Steam Engines; Trans. Am. Soc. Mech. Engrs., 1881; JOUR. FRANKLIN INSTITUTE, May, 1881.

ify the methods of expansion and of expenditure of heat so greatly that the ratio of expansion for maximum efficiency, in unjacketed engines, rarely exceeds $\frac{1}{2}\sqrt{P}$, where P is the gauge pressure in pounds per square inch, although its value would otherwise be, often, several times greater. It was also shown that these modifying conditions very differently affect different kinds of steam engine and different engines and also individual engines, at various pressures and piston speeds.

It is now proposed to show more fully what is the behavior of steam in the steam engine cylinder and to exhibit the form taken by the expansion line; to give values of mean pressures obtained under various conditions and, finally, to show the method devised by the author for laying down curves of efficiency, and to show what conclusions follow from their study.

It has become plainly evident that in no case, in steam engines as to-day constructed, can the expansion line or the curve of mean pressures be such as would be obtained in a non-conducting cylinder. Steam must always be more or less condensed at the begining, and must always carry away heat by its re-evaporization at the end of the stroke. The steam jacket checks the first operation, but accelerates the last, and, with wet steam, may scarcely decrease the evil that it is designed to prevent.

It is now equally evident that "isothermal" expansion is not likely to be met with in the steam engine, although the hyperbolic expansion line is often approximately followed. This, however, with steam and other vapors, is very different from isothermal expansion; the latter is physically impossible with wet steam, and the isothermal expansion of steam, initially dry, but saturated, would leave the vapor superheated, although at initial temperature when exhausted.

To produce this last condition heat must be furnished by the steam jacket sufficient not only to do the work of expansion, as measured by the indicator, but also a large additional quantity which is demanded to do the "internal work" of overcoming molecular attractions, *i. e.*, to supply the "latent heat of expansion" at constant temperature.

On the other hand, hyperbolic expansion of dry steam would leave the fluid superheated, also, but not at the initial temperature; while steam containing about its own weight of water, if expanded hyperbolically, would be exhausted, with the proportions of the mixture

unchanged, and with its temperature reduced to that due the lessened pressure.

Isothermal expansion lines of a *mixture* of steam and water are simply lines of uniform pressure, and could only be obtained in the steam engine by some system of heating the mass at just the rate necessary to evaporate the water as rapidly as steam might be required to keep up the pressure while the piston was moving forward—a state of things never met with in practice.

The expansion line is not only modified in position and in form by the conductivity of the cylinder, but, also, although far less seriously, by the quantity of water contained in the mass of fluid at the instant of closing the expansion valve.

The adiabatic curve may be closely represented by a regular curve of the hyperbolic class, $p_1 v_1^n = p v^n$, the exponent n varying with the proportions of steam and water in the mixture at the commencement of the expansion, which is assumed to take place in a nonconducting cylinder. Zeuner finds the value of n to be nearly $n = 1{\cdot}035 + 0{\cdot}1\,x$, x being the proportion of steam present.

Table I, appended, gives the values of the ratio of mean pressure to initial pressure, $\frac{p_m}{p_1}$, for various mixtures from steam 1·00, water 0, to steam 0·50, water 0·50, assuming the formula to be practically accurate within that range.

With these are given the adiabatics for superheated steam, $n = 1{\cdot}333$.

Table II gives the values of $\frac{p_m}{p_1}$ for steam expansion in a jacketed metal cylinder, in which it is kept just dry and saturated by heat from the jacketed sides and ends; the values for wet air compressed in air compressors, in which n is frequently found to be 1·2; and for peculiar cases in actual steam engines in which leakage or re-evaporation, or both, raise the terminal pressures greatly, giving $n = 0{\cdot}50$, $n = 0{\cdot}75$.

It is, as yet, impossible to predict which of these curves will be found in ordinary engines, and the engineer is compelled to rely entirely upon the "indicator" for information of this character; this instrument gives him a more or less exact graphical representation of the changes of pressures and volume throughout the stroke. The greatest possible variety of curves are found to occur in such cases,* but

* An indicator diagram lying before the writer gives $n = 1{\cdot}001$ at the beginning of

they approach the adiabatic more nearly, as the steam is drier and, as the speed of piston is increased, rarely departing far from the common hyperbola in good engines. Perfectly dry or superheated steam in very fast running engines gives a curve most closely approaching the adiabatic curve, but the deviation is more marked as the speed of engine is decreased, and as the amount of moisture in the steam, initially, increases. The limit may be taken as $pv = p_1v_1$, on the one side, and to $pv^{\frac{1}{2}} = p_1v_1^{\frac{1}{2}}$ on the other, the latter being the rare case sometimes met with of an unjacketed engine working at a piston speed below 50 feet per minute (under 15 metres), and with a high ratio of expansion, while the former is a very usual limiting value with well-constructed jacketed engines at good speed.

These curves take the form here indicated, the exponent n becoming less than unity—instead of greater, as in all cases of adiabatic expansion, and usually of expansion in well-jacketed cylinders—in the following manner: Consider a steam cylinder unjacketed and worked at some extremely low speed of piston.

At the instant before the steam enters either end, that end is exposed to the chilling action of vapors forming on its warm surfaces at the pressure of the exhaust, while the opposite end is just releasing a charge of steam which has been expanded down to some pressure considerably below that of prime steam; the expansion of which charge and the re-evaporation of moisture present, chills the surface of the cylinder on that side of the piston also.

Now, the steam entering from the boiler meets the cooled metal surfaces of piston and cylinder head and condenses upon them until they attain boiler temperature, or until the closure of the steam valve checks this operation, and the charge then expands as the piston moves forward with its very slow motion, uncovering, gradually, surfaces which had been cooled by the exhaust on the other side of the piston. The expanding steam, meeting these cold surfaces, surrenders a part of its heat until they are brought fully up to the temperature of the steam, by flow of heat into the interior of the metal, after which time they, in turn, give up heat again to the steam, producing

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the stroke, $n = 0{\cdot}94$ at the midde and $n = 0{\cdot}89$ at the end. The compression line starts with $n = 1{\cdot}52$ and varies thus, $n = 1{\cdot}29$, $n = 0{\cdot}96$ to the end, where $n = 0{\cdot}77$, showing that the mean temperature of the surfaces in contact with steam is above that of the vapor during the first half of the period of compression, and below that of the fluid during the second half.

re-evaporation of the water present and giving a pressure exceeding that which would be due the same volume of expanding dry steam.

The total effect is evidently determined by the varying temperature of the cylinder walls, the temperature of exhaust and the dryness of the steam supplied.

As the piston moves forward, all parts of the cylinder hotter than the mean temperature of exposed surfaces, and all the water present, surrender heat which causes the production of steam and a tendency to increase the normal pressure. The colder surfaces constantly uncovered by the piston absorb heat, produce condensation of steam and tend to reduce pressure below the normal. As the one or the other of these effects predominates, the expansion line rises above or falls below that obtained by adiabatic expansion.

These effects are sometimes modified and rendered less observable by the occurrence of leakage—usually and principally through the steam valves—but where, as is frequently the case, the actual expansion line first falls below and then rises above the adiabatic or hyperbolic curve it may be probably safely assumed that leakage does not occur to any important extent.

Where the steam contains much water the expansion line in actual engines often, if leakage occurs, lies entirely above the curve of Marriotte, the value of n being less than unity. In other cases, the line may fall under the hyperbola at the beginning but rise far above it toward the end of the expansion, giving a curve more nearly parabolic in appearance and with a mean value of n less than unity. This increase of area of indicator diagram rarely, if ever, denotes any important gain of work done by re-evaporation.

The values of $\frac{p_m}{p_1}$ given in the tables are plotted on Plates I and II and from the latter a better idea can be obtained of the method of variation of mean pressure and of work done with variation of the ratio of expansion and a better notion of the relation of these several curves than can be gained by the study of the tabled quantities. A comparison of the engraved curves will be both interesting and instructive to the engineer. It will be observed that the adiabatics are often so nearly coincident with the hyperbolic curve, which is also the isothermal for gas, that the assumption of hyperbolic expansion, within usual ranges of expansion, whatever the mixture, is not likely to lead to serious error in estimating work. They, however, correspond to vastly different quantities of heat expended.

The tabulated values of mean pressures for adiabatic expansion and the curves exhibiting them to the eye are evidently entirely useless for the purposes of the engineer except in the few, possibly extremely rare, cases in which the modifying action of the metallic cylinder may be neglected. Such cases can only be met with, if met with at all, in the study of efficiently jacketed compound engines or of very fast running, unjacketed, non-condensing engines.

Experiment shows that superheating steam is not always very effective as a preventive of exhaust waste and that the ratio of expansion at maximum efficiency is practically the same, in unjacketed engines, whether steam is dry or superheated. The advantage undeniably obtained by the use of superheaters is, therefore, in such cases, evidently only due to the increased range of temperature of the working fluid and not to increased efficiency of engine as a machine simply.

The pressures given for a non-conducting cylinder are never exactly —and seldom, perhaps, even approximately—accurate for cases which are familiar to the engineer in practice.*

But although these curves of mean pressure are valueless, usually, for for direct application, the engineer will find them useful—indispensable, in fact—in the construction of probable mean pressure curves for proposed engines; and by properly applying them in the manner to be described, as devised and made use of by the writer, he may obtain very accurate and practically valuable Curves of Efficiency for any given class of engines; and he may then conveniently and satisfactorily solve all problems, arising in his practice, relating to efficiency of fluid, of engine or of capital expended.

Referring to Plate B, suppose a pound, a cylinderful, or other unit

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* Rankine, it is true, when investigating the theory of the condensing engine and comparing his results with those obtained by Wicksteed from the Harvey & West Cornish pumping engine, at Old Ford, finds a practical identity. This is, however, an accident due to errors of treatment of the case. He does not take into account a loss of heat by avoidable forms of exhaust waste which often amounts to a very large proportion of all heat expended and is rarely less than twenty per cent.; he omits jacket expenditure which is often ten per cent. more; but the values of n and p_1 taken in the calculations and which are inaccurate, as is now known, are such as just to compensate these errors of the expense account and to produce substantial accordance of calculated with experimental results.

A good Cornish engine, it should however be said, is peculiarly well adapted by its effective steam-jacketing, its singular distribution of steam and its quick "indoor" stroke, to secure a maximum ratio of economical expansion and approximation of actual to ideal conditions.

of quantity of steam and water to be drawn from the boiler, carrying —we will assume with Grashof—10 per cent. its total weight of water, 90 per cent. being saturated steam, and to have a pressure which may be called 1·00 When separated from the boiler and carried into the cylinder it will retain the pressure 1·00 and, worked at full stroke, will do the work 1·00.

If supplied with additional heat until completely dry, the work becomes 1·11 at full stroke and, if worked at different ratios of expansion, such steam will give a series of mean pressures represented by the Curve of Efficiency, A_1, Plate III, as obtained from the expansion curves whose equation is $pv^{1.135}$ = constant, provided expansion occurs in a non-conducting cylinder where no condensation can occur except such as is due to performance of work.

Expanded wet, as drawn from the boiler, the mean pressures of Curve *B*—from $pv^{1.125}$ = constant, which is deduced by Zeuner for $x = 90$—are proportional to the work done by the mixture if worked without change of proportion other than occurs by production of work.

If, again, the same weight were drawn from the boiler at the pressure assumed and in the same proportions—steam 90, water 10—and if, on entering the cylinder, initial condensation should double the quantity of water present, the work at full stroke would be ·90 and the mixture would, at other ratios of expansion, the proportion remaining unchanged, give relative quantities of work measured by the ordinates of Curve C: $pv^{1.116}$ = constant. It now contains steam 81, water 19.

Similarly, the proportion of water present being increased by initial condensation from the original amount carried out of the boiler, so as to reduce the work of unity of weight to ·80, ·70, ·60, ·50, etc., at full stroke, the curves of efficiency become as shown in Plate III, Curves D, E, F, etc., successively, down to the base-line where condensation has become complete and the work of expansion of the water may be neglected.

Such are the curves of efficiency, of work and of mean pressures to be obtained where steam is expanded in a non-conducting cylinder. They are easily deduced and easily constructed and, by reference to Zeuner's formula, the engineer can determine them with a satisfactory degree of accuracy for all cases which are likely to arise in his practice.

Now, studying the behavior of steam in a metallic cylinder, we

shall find vitally different conditions and results; but given the law of variation of composition of the mixture with change of point of cut-off, or of ratio of expansion, it is, nevertheless, not only practicable but easy to determine curves of efficiency and to deduce values of the best ratio of expansion for any given case.

Where direct experience can be resorted to, to determine the cylinder condensation, it is easy, as will be shown hereafter, to obtain *exact* results when seeking the ratio of expansion at maximum efficiency of fluid, of engine or of capital or in the solution of a rarer case which requires that the point of cut-off which gives most work for a given expenditure on the whole plant be determined.

In the actual engine, steam entering from the boiler—at the instant of starting the piston forward—consists of a mixture of steam and water, of which the proportions are determined by the character of the boiler-steam and the amount of initial condensation. As the piston moves forward, this proportion becomes independent of all external conditions at the instant of the closing of the steam valve. From this point on, the interchange of heat between the steam and the surrounding walls of the cylinder produces a continuous change of proportion until the exhaust-valve opens.

Thus, assuming steam to enter at a pressure 1·00 and to contain 10 per cent. water, its Curve of Efficiency* starts on Curve *B* and gradually shifts from curve to curve—as seen on the plate, Curves *K*, *L* and *M*—more or less rapidly, as cylinder condensation takes place to a greater or less extent, the real Curve of Efficiency usually crossing *C*, *D*, *E*, etc., and taking the general form indicated by lines *K*, *L*, *O* and *P*.

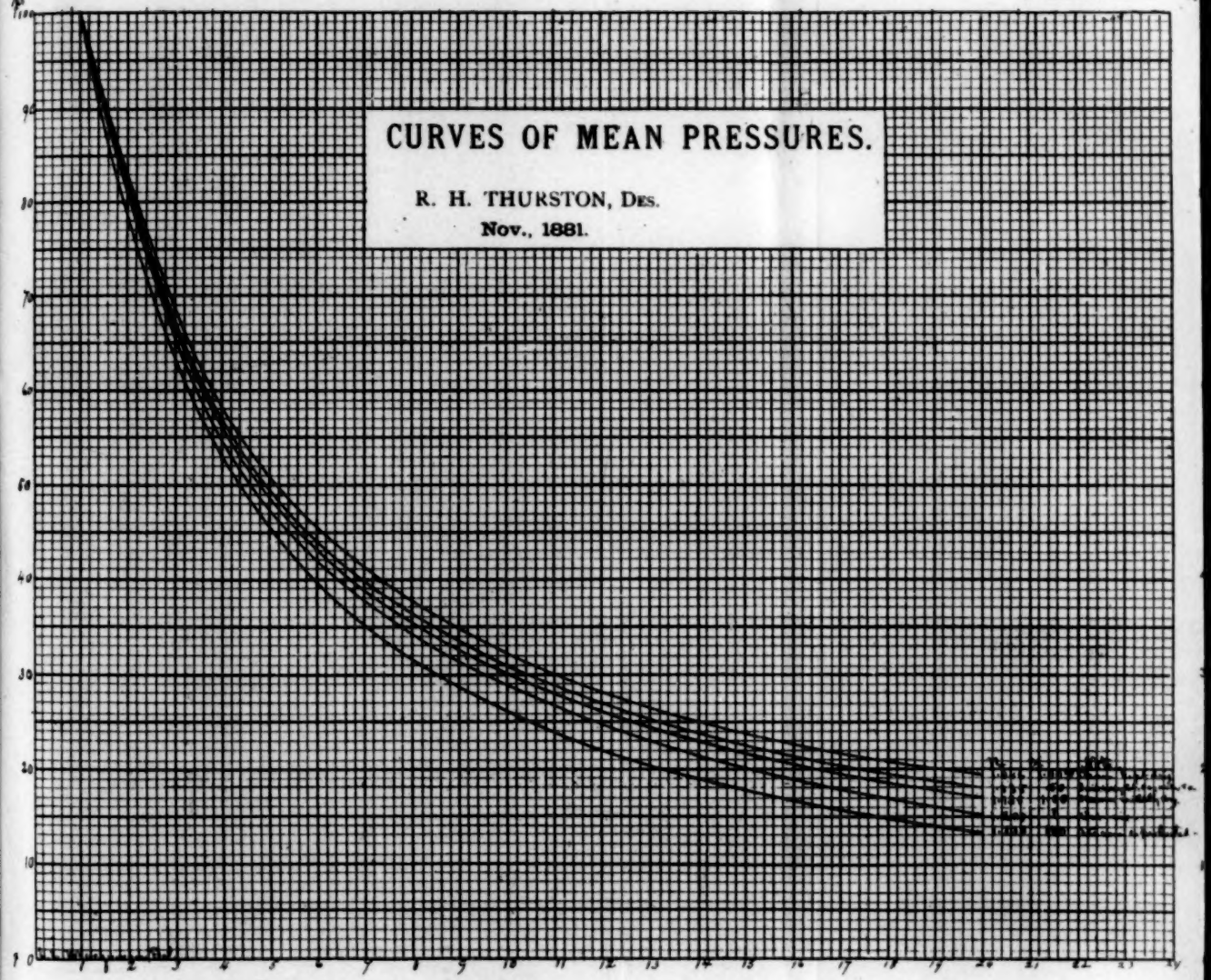
With considerable expansion and wet steam, the expansion line may again rise during any one stroke, by re-evaporation, toward the end of the stroke to such an extent as to somewhat increase the mean pressures, but this case is, apparently, not a very common one.

The amount of that condensation is, evidently, a function of the ratio of expansion in every engine and the writer has been accustomed to take it as varying approximately as some power of r.

Lines *K*, *L* and *M*, which are presented simply in illustration, represent, respectively, the Curves of Efficiency when the total loss by

* The Curve of Efficiency and of Mean Pressures must not be confounded with the Expansion Line representing the varying relations of pressure and volume during the stroke.

PLATE I.



STEAM IN THE STEAM ENGINE CYLINDER.

PLATE II.

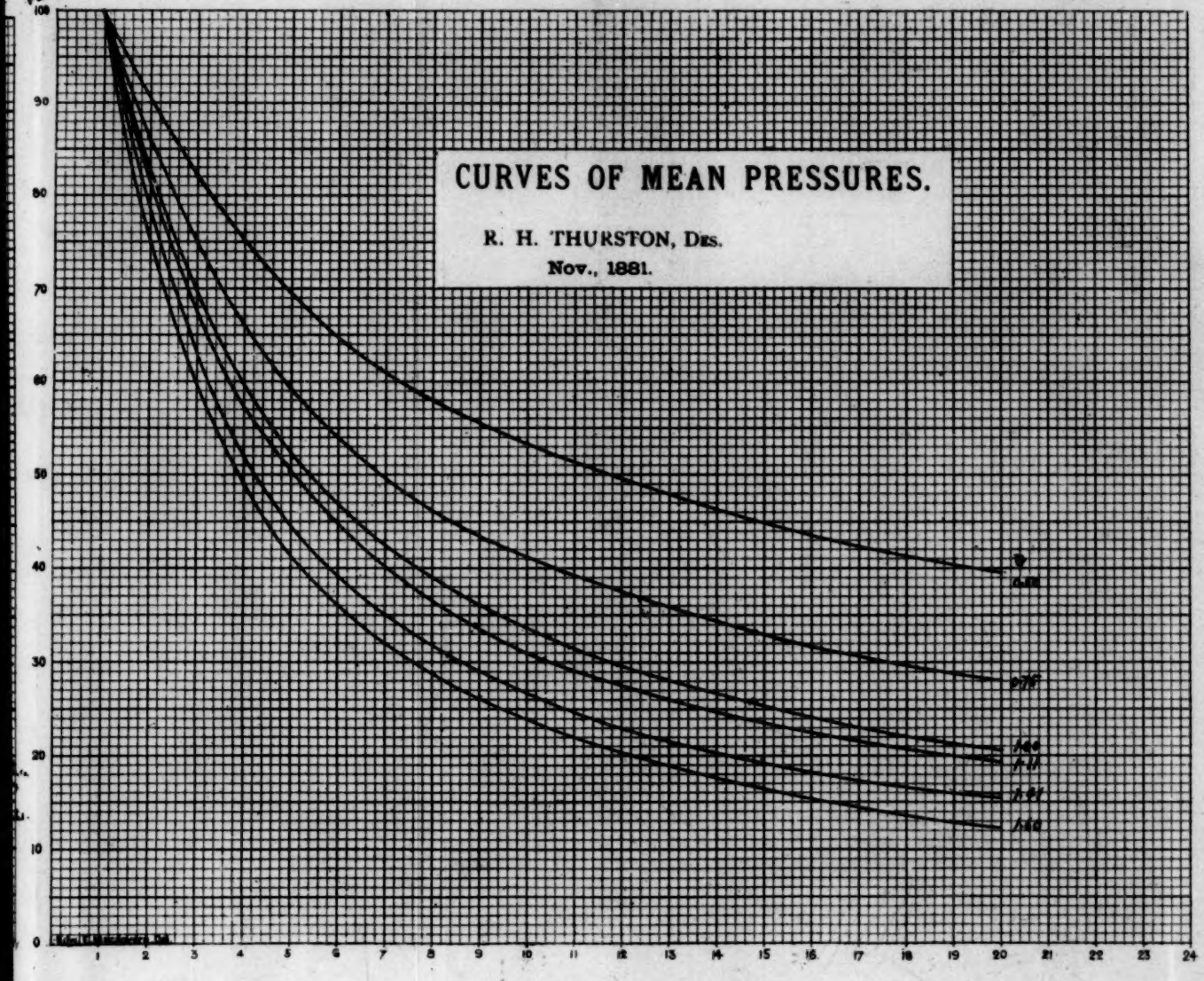
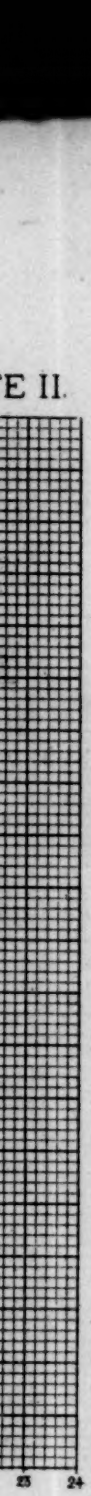
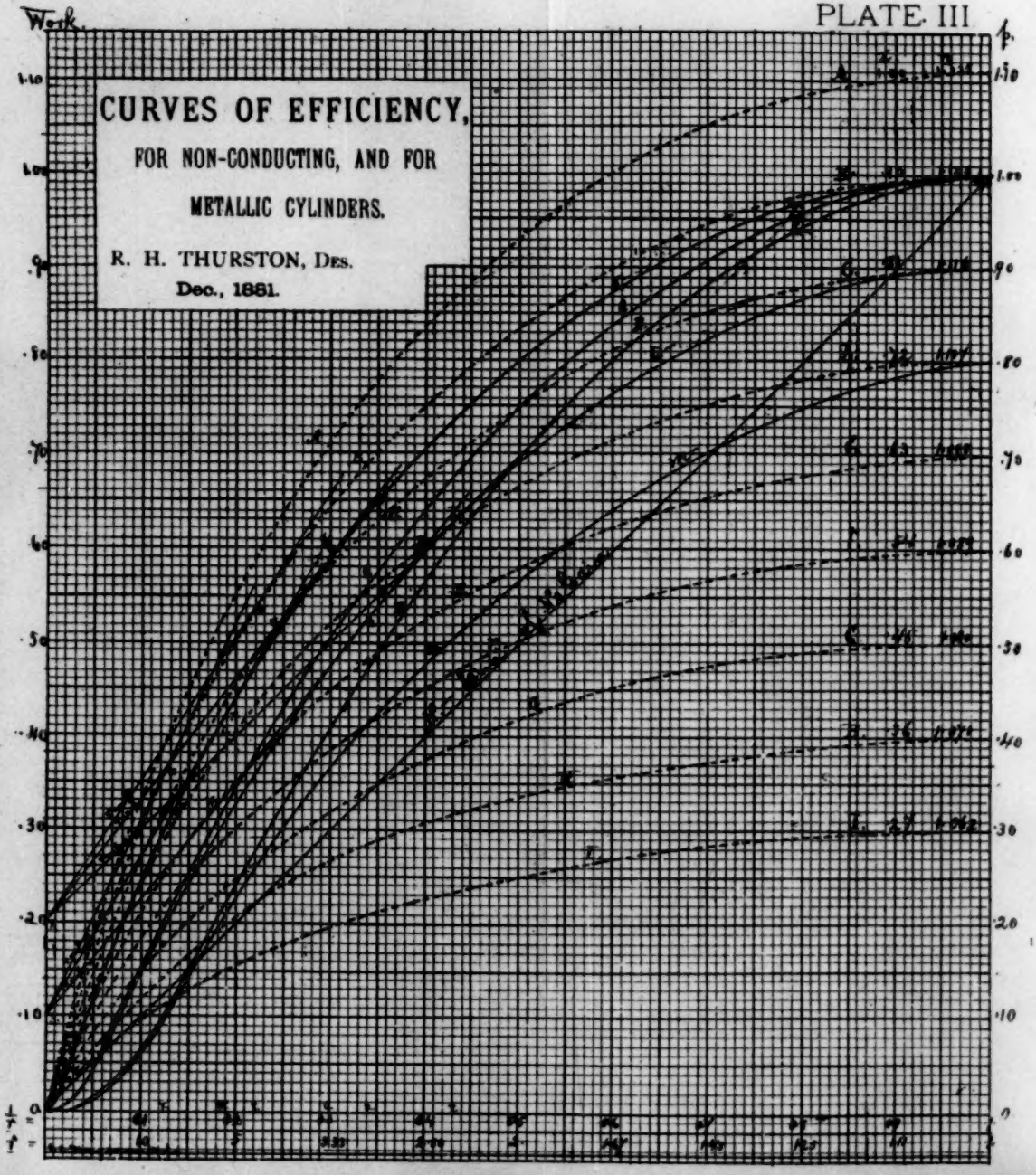


PLATE III

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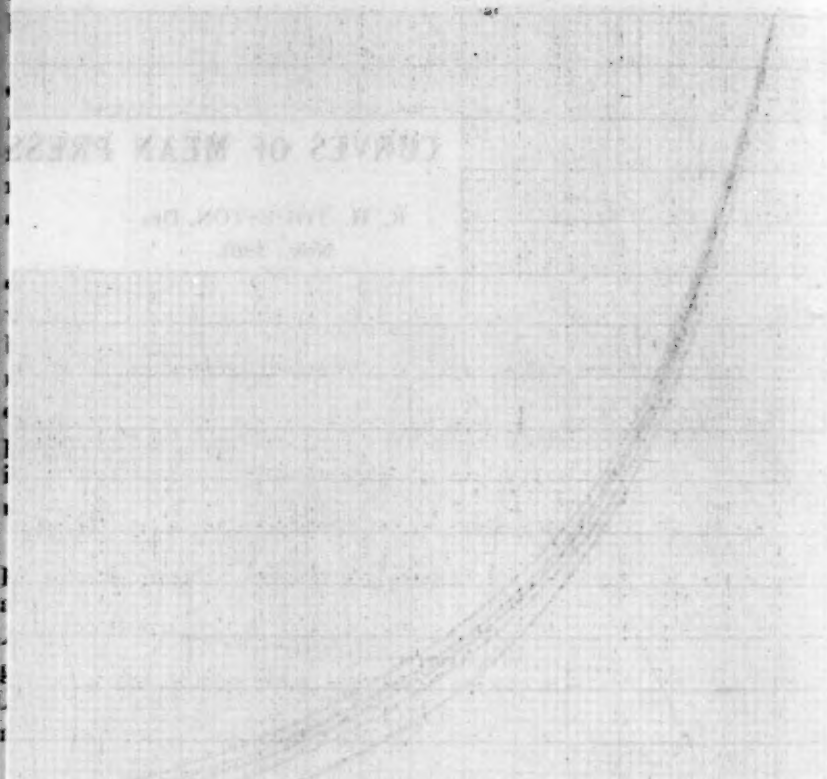
CURVES OF EFFICIENCY,
FOR NON-CONDUCTING, AND FOR
METALLIC CYLINDERS.
R. H. THURSTON, Des.
Dec., 1881.



cylinder condensation, h_c, varies as $\sqrt{r}$, and when $h_c = 0{\cdot}1\sqrt{r}$, $h_c = 0{\cdot}2\sqrt{r}$, $h_c = 0{\cdot}3\sqrt{r}$, values which will be found not uncommon in engineering practice. The abscissas of the curves are, as before, measures of weights of steam used.

If, in any case, condensation were so to vary that no gain should be derived from expansion—and such cases are, within a limited range of expansion, sometimes nearly approximated to—the curve of efficiency would become a straight line, *N*, the "Line of Constant Efficiency," Plate III.

The curves *O* and *P* are obtained by altering the vertical scales of *L* and *M* so as to give them a common initial point with *B* and *K* at $p = 100$ and thus enabling the reader to compare the differences of form of the several lines and of the two kinds of curve more satisfactorily. It will be seen, also, on comparing the second of the two kinds of curve with those derived from experiment on working engines, and to be presented in a succeeding paper, that the Curve of Efficiency here obtained by induction is of precisely the same character as that given by direct experiment.

To construct the theory of these cases of non-adiabatic expansion, the writer has taken the following method: We may take two distinct cases: (1.) That in which, as when the cylinder is unjacketed and unprotected against radiation and the ratio of expansion small or the temperature of exhaust very low, so little re-evaporation occurs that it may be neglected; (2.) That in which, as in jacketed cylinders with wet steam and considerable expansion, nearly all condensation occurs before the point of cut-off is reached and re-evaporation takes place throughout the remainder of the stroke.

Case 1.—It is shown in all works on thermo-dynamics, that the form of the expansion line of steam, either dry and saturated or containing liquid water, may be obtained for adiabatic expansion from approximate equations of the form $pv^n = p_1 v_1^n$; $p_2 = p_1 r^{-n}$. Since loss of pressure occurs in the metallic cylinder by a transfer of heat, taking place by initial condensation and re-evaporation, and since the amount of this loss is determined, in any given cylinder, by the magnitude of the ratio of expansion, we shall obtain for the value of the pressure in the latter case $p_1^1 = p_1 \left(\frac{v_1}{v}\right)^n \left[1 - f(r)\right]$

The values as well as the form of this function of r, $f(r)$, above, are not yet ascertained. The function is evidently determined by the area

of cylinder surface, time of exposure, variation of temperature of surface and quantity and character of the working steam. The form of $f(r)$ is probably transcendental and it may be exponential. The writer has found that for the ordinary values of the ratio of expansion, as an approximation, we may take $f(r) = ar^m$; m being taken constant.

In this expression a has a value which is determined by the state of the steam at entrance into the cylinder and, if the steam is initially dry, is connected with the exponent n by some definite relation. The value of m is dependent upon the character of the engine and the method of its operation, so far as they determine the rate of variation of the proportions of steam and water during expansion. Given the values of n and of a, m becomes determinable. We have

$$\frac{p_2}{p_1} = r^{-n} - ar^{m-n};\ m = \frac{\log.\left[\frac{1}{a}\left(r^{-n} - \frac{p_2}{p_1}\right)\right]}{\log.r} + n,$$

where p_2 is the terminal pressure, a quantity always known when r is known by experiment.

The equation for the expansion line, the working substance being enclosed in a metallic cylinder, is then

$$p = p_1\left(\frac{v_1}{v}\right)^n\left[1 - a\left(\frac{v}{v_1}\right)^m\right]$$

The work done by expansion is

$$\int_{v_1}^{v_2} pdv = p_1 v_1^n \int_{v_1}^{v_2}\left[1 - a\left(\frac{v}{v_1}\right)^m\right] v^{-n}dv$$

The net work is $Wn = p_1'v_1 + \int_{v_1}^{v_2} pdv - p_b v_2$

in which p_b is the back pressure plus friction and useless resistance.

The terminal pressure is given above. Making $r = 1$, we obtain from that equation $p_2 = p_1(1-a) = p_1'$, showing that p_1 is not the initial cylinder pressure, p_1', but the pressure which the same weight of steam would have given if working at the same volume and without condensation in the same cylinder; p_1 exceeds p_1' in the ratio $1:1-a$; which ratio measures the working relative values of the same mass of steam with and without condensation.*

Integrating the expression for net work done during expansion,

*If x is the "dryness-fraction" of the steam when worked at full stroke, it having been dry when drawn from the boiler, $p_1' = p_1 x$; $x_1 = \frac{p_1'}{x}$.

$W_e = \int_{v_1}^{v_2} p dv - p_1 v_1^n \int_{v_1}^{v_2} v^{-n}\left[1 - a\left(\frac{v}{v_1}\right)^m\right] dv$, we obtain

$$W_e = \frac{p_1 v_2}{1-n} r^{2-n} - \frac{p_1 v_2}{1-n} r^{-1} - \frac{a p_1 v_2}{m-n+1} r^{m-n} + \frac{a p_1 v_2}{m-n+1} r^{-1} - p_b v_2 =$$

$$\frac{p_1 v_1}{1-n} r^{1-n} - \frac{p_1 v_1}{1-n} - \frac{a p_1 v_1}{m-n+1} r^{m-n+1} + \frac{a p_1 v_1}{m-n+1} - p_b v_1 r.$$

while the total useful work per stroke is $W_u = p_1' v_1 + W_e$.

In this analysis the effect of re-evaporation is entirely neglected as unimportant. It will usually be found, if the writer may judge from his own observation, that it has no important effect in increasing efficiency or in rendering the treatment here adopted inaccurate—except in the determination of final pressures as shown below—for such cases as are included under Case 1.

The equation of these "Curves of Efficiency," as the writer would call them, for adiabatic expansion is

$$y = p_1 \frac{n - r^{1-n}}{n - 1} x.$$

The equation for the present case is

$$y = \left[\frac{r^{2-n} - 1}{1-n} - \frac{a r^{m-n+1} - a}{m-n+1} + 1\right] p_1 x, \text{ nearly.}$$

The mean pressure is then

$$p_m = \frac{p_1 r^{2-n} - p_1 r^{-1}}{1-n} - \frac{a p_1 r^{m-n} - a p_1 r^{-1}}{m-n+1} + p_1' r^{-1}.$$

and the mean effective pressure is

$$p_e = \frac{p_1 r^{2-n} - p_1 r^{-1}}{1-n} - \frac{a p_1 r^{m-n} - a p_1 r^{-1}}{m-n+1} + p_1' r^{-1} - p_b.$$

The mean effective pressure and the work of the machine are maxima when the back pressure, p_b, is fixed, when

$$p_2 = p_1 r^{-n}[1 - a r^m] = p_b,^*$$

provided, as assumed, re-evaporation may be neglected. Then

$$r^{-n} - a r^{m-n} = \frac{p_b}{p_1}.$$

The Ratio of Expansion for Maximum Efficiency is that which satisfies the above equation, and the fraction of stroke completed at the instant of closing the steam valve is $\frac{1}{r} = c$ and its value is that

* In fact, however, re-evaporation—the effect of which is not in such cases usually found to be appreciable in increasing efficiency—prevents the fall of the terminal pressure quite to the value $p_2 = p_b$.

which gives $c^n - ac^{n-m} = \frac{p_b}{p_i}$

Examining the last two equations, it is seen that the smaller the valve of a and the less the values of $m-n$, the more nearly does the value of r approximate to the ratio of initial terminal pressures. It is also evident, both from the analysis and from the graphical construction to be described, that the ordinary variations in the values of n, due to presence of water, do not practically alter the ratio of expansion at maximum efficiency in non-conducting cylinders, as this variation of the exponent rarely exceeds the limits 1·135, 1·125 for such cases.

The steam from well-proportioned and well-managed steam boilers seldom entrains even ten per cent. water. The experiments of the writer* and others give an average of 3·2 per cent. water in steam from boilers not fitted with superheaters, and Hallauer finds less than five per cent., while Isherwood's experiments confirm these figures.

For usual cases, therefore, $n<1{\cdot}135$ and $n>1{\cdot}130$, no condensation taking place.

Working steam in metal cylinders, however, the case is, as has now been shown, radically different, and the efficiency of the engines and the best ratio of expansion are very greatly altered by variation in both the initial proportion of water entrained and the rate of cylinder condensation. The value of a varies from 0·1 to some much higher figure and the value of m varies from less than unity to at least two. It thus happens that the ratio of expansion, at maximum efficiency, is sometimes reduced to less than one-half that which should be obtainable in non-conducting cylinders.

It is further evident that the great loss of heat thus met with may also be experienced even with superheated steam, as heat may be transferred by conduction and convection as by initial condensation and final re-evaporation. Experiment has shown in fact, as already indicated, that, in some cases, with unjacketed condensing engines, the ratio of expansion at maximum efficiency is not much increased in value by superheating and, hence, that the increased economy noted comes, not from increased efficiency of the machine, but simply from increased range of temperature of working steam and reduced back-pressure; the cylinder-condensation is prevented but the loss of heat

* American Institute Report on Steam Boilers, 1871; Journal Franklin Institute, 1872.

still occurs, the transfer being effected by a different process.* The above remarks relative to saturated steam are therefore applicable to all fluids used in heat-engines. In fact, in gas-engines, where all vapor is enormously superheated, the loss by exhaust waste thus caused is found to be, sometimes, twice as great as the total quantity of heat which would be demanded were the cylinder made of non-conducting material.

The following cases, illustrating the results of this method of treatment of Case 1, as applied to several selected examples such as are met with in ordinary practice, are given as exhibiting the usual range of values of the quantities involved in the preceding equations.

Character of Engine.	p_1	p_b	a	m	n	r_e
I. Non-condensing engine,	100	20	0·2	1·5	1·115	4·5
II. Condensing, unjacketed,	40	5	0·2	0·5	1·115	2·5
III. " compound, jacketed,	60	6	0·1	1·1	1·125	6·0
IV. " " "	100	5	0·1	0·0	1·135	10·0

The real ratio of expansion is given in all cases where it is not expressly stated that the value given is that of the apparent ratio. In the first three of the above cases, the steam is taken from the boiler dry; in the last case it is so far superheated that it expands as dry steam, all cylinder-condensation being prevented.

Case 2.—The second case is very frequently met with in practice. In this case, initial condensation ceases with the closing of the expansion valve, and in jacketed engines or in engines worked with considerable compression,† even before, and re-evaporation occurs throughout the whole period of expansion. Then, if $b = 1-a$; $a+b = 1$; and if q represent the new exponent, we may write, for the net power delivered,

$$W_a = br^q p_1 v_1 \frac{n-r^{1-n}}{n-1} - p_b r v_1;$$

$p_1 v_1$ measuring, as before, the work obtainable from the same weight of *dry* steam, up to the given point of cut-off, when working at the same ratio of expansion and when, therefore, $p_1' v_1 = bp_1 v_1 = (1-a)\, p_1 v_1$ as in the first case. This becomes a maximum when

* The now familiar expedients of high speed of piston and of "compounding" engines seem the most successful means yet adopted to check this waste in modern engines. Superheating is also most effective in compound engines.

† Note on The Expansion of Steam and Regulation of the Engine, by R. H. Thurston, Trans. Am. Soc. Mech. Engs., 1881; JOURNAL FRANKLIN INSTITUTE, Oct., 1881.

$$r^{q-1} - \frac{q+1-n}{qn} r^{q-n} = \frac{n-1}{bqn} \frac{p_b}{p_1}.$$

The mean effective pressure is

$$p_e = br^{q-1} p_1 \frac{n - r^{1-n}}{n-1} - p_b,$$

and the equation of the Curve of Efficiency is, for this case of non-adiabatic expansion,

$$y = b^2 r^{2q-2} \frac{n - r^{1-n}}{n-1} p_1 x.$$

For the case of nearly hyperbolic expansion, which is a common one for this class of engines, $W_n = bp_1v_1(1+\log_e r)\, r - p_b r v_1$, nearly; which is a maximum when

$$\left[\left(q(1+\log_e r) \right) + 1 \right] r^{q-1} = \frac{p_b}{bp_1}.$$

The mean effective pressure is $p_e = bp_1 (1+\log_e r)\, r^{q-1} - p_b$.

The value of q varies from 0 to 0·3, being greatest with most efficient engines.

The ratios of expansion for maximum efficiency are those which satisfy the above equations. The terminal pressure is nearly

$$p_2 = \frac{p_b}{br^q}.$$

The following are corresponding values of a, b and n:

a	0·00	·10	·20	·30
b	1·00	·90	·80	·70
n	1·135	1·125	1·115	1·105

The Curve of Efficiency for this last case has the form

$$y = b^2 p_1 (1 \times \log_e r) r^{2q} x.$$

The value of q lies between 0 and $\frac{1}{2}$ in most cases. The consumption of steam and cost of power in these cases is measured by the volume introduced at the pressure p_1, precisely as with the non-conducting cylinder.

Referring once more to the set of curves of efficiency, Plate III, we may deduce the same conclusions from graphical construction and obtain results far more easily and rapidly. Selecting values of $\frac{p_b}{p_1}$ such as are often obtained with non-condensing and with condensing engines, respectively: $\frac{p_b}{p_1} = ·20$; $\frac{p_b}{p_1} = ·10$, we may determine ratios for maximum efficiency of engine thus: From the points ·20 and ·10

on the axis of ordinates on the scale measuring total work per stroke, draw lines tangent to the several curves, as RT, RV, SV, SW, etc., etc. The points of tangency being found, the values of their abscissas measure the quantities of steam to be used per stroke to give maximum engine efficiency, since the ordinate of any point divided by the abscissa is a measure of the ratio of work done to steam expended in doing it and, for the assumed back-pressures, the *net* amount of work per unit's weight of steam is a maximum at the points just identified.*

On making the construction it will be found that these maxima are found for very nearly the same values of abscissa and, therefore, for the same ratio of expansion, nearly, whatever the dryness-fraction of the steam used in the non-conducting cylinder.†

But, drawing tangents RK, RY, SX, SZ, etc., to curves K, C and M, to determine the best ratios for the metallic steam cylinder, values are formed for r far removed from those just obtained for the non-conducting cylinder and also differing among themselves greatly with the proportion of water present. In the cases shown on the plate, the ratio for the non-condensing engine is decreased to two-thirds and for the condensing engines to less than half that found best for the non-conducting cylinder.

It is to be remembered that the quantity of steam used per stroke, although in direct proportion to the distances "followed" by the steam up to point of cut-off in the non-conducting cylinder, may be in widely different proportion with the metal cylinder. In the latter it varies from nearly an equal proportion at full stroke to, often, a double proportion at high ratios of expansion.

This will be seen more clearly when studying actual engines, as will be shown in a succeeding paper, in which working figures will be given.

The writer concludes from what has been above shown:

(1.) That the work done in a non-conducting cylinder, the fluid expanding adiabatically, varies so little with the proportion of water present that this variation may be neglected by the engineer and he may assume the performance of work to be such as would come of

* This principle was pointed out by Rankine. See Miscellaneous Papers, p. 295, and Shipbuilding, appendix.

† Note that, for curve C, D, etc., the same values of $\frac{p_b}{p_1}$ must be used and the base lines have ordinates, respectively, 0.9, 0·8, etc., those drawn for the curve B.

nearly hyperbolic expansion, while the heat expended may be exactly calculated from the quantity of work, when the latter is known.

(2.) That, in metal cylinders, the work done at any given point of cut-off is nearly the same as in the non-conducting cylinder, but that the quantity of heat and of steam expended in doing it are increased, and usually very greatly increased, by cylinder-condensation if saturated or wet steam is used, or by other methods of transfer of heat to the exhaust and by consequent waste, if superheated steam or other gaseous working fluid is employed.

(3.) That the ratio of expansion at maximum efficiency would be practically unchanged by ordinary variations in the proportion of water entrained with the steam, if it were worked in a non-conducting cylinder, and the value of that ratio, r_e, is very nearly $\frac{p_1}{p_b}$, the quotient of initial pressure divided by the sum of the cylinder back-pressure and other useless resistances.

(4.) That the ratio of expansion at maximum efficiency, when steam or other fluid expands in a metallic cylinder, is more seriously affected by the introduction of water entrained by the steam, and this difference is increased and usually is made very serious by the occurrence of cylinder-condensation, or other method of transfer of heat to the exhaust, when a metal cylinder is used. This ratio becomes, in this case, much less, usually, than $r_e = \frac{p_1}{p_b}$.

(5.) That the quantity of fluid used per stroke in the non-conducting cylinder is in direct and exact proportion with the volume of the cylinder open to the supply-pipe at the instant of closing the expansion-valves and is measured by $\frac{1}{r}$, the reciprocal of the ratio of expansion.

(6.) That the volume of boiler steam worked per stroke in the metal cylinder is *not* in direct proportion to volume of cylinder open to steam at the point of cut-off; but that it is often very greatly in excess of the latter quantity and that it becomes greater, as the ratio of expansion is increased, indefinitely.

(7.) That the ratio of expansion is not a gauge of the volume of steam demanded from the boiler and paid for by the proprietor of the apparatus when the metal cylinder is employed; but that the volume

of steam used or quantity of heat demanded must very greatly exceed the proportion $\frac{1}{r}$ in nearly all actual engines.

(8.) The Curve of Variation of Efficiency above traced, of which the abscissas measure varying quantities of steam used in a given steam cylinder, while the ordinates are proportional to the quantities of work done by those amounts of steam, is a curve of entirely different character and form, and often widely different in location, from the Curve of Adiabatic Mean Pressures or other curve of mean pressures exhibiting the work done by various quantities of steam expanding under given fixed conditions in a non-conducting vessel.

(9.) That no predetermination of the efficiency of any proposed engine, whether of fluid, of machine or of capital, can be made unless the true curve of efficiency can be obtained for the assumed case.

(10.) That the most certain and the most satisfactory solution of any problem of efficiency will be that obtained by first securing the elements of the Curve of Efficiency from actual engines operated in the manner proposed for the case taken.

(11.) That, having obtained by experiment upon any engine, the "Curve of Efficiency," as defined by the writer, the efficiency of fluid, of engine and of capital expended to do a given amount of work, and the quantity of work to be obtained most cheaply from a given engine, may all be obtained for any given set of conditions, and the ratio of expansion, at maximum efficiency, of fluid, of engine and of capital and the ratio of expansion which, with a given plant, gives most work for a dollar of expense of operation, may all be determined with a degree of exactness only limited by the magnitude of the errors of observation.

(12.) That the necessity of following the direction of improvement pointed out and entered upon a century ago, by Smeaton—the protection of the working fluid from losses of heat, by surrounding it with non-conducting surfaces—constitutes the most imperative of all demands to-day made upon the mechanical engineer who may be engaged in designing steam engines.

Hoboken, N. J., Dec., 1881.

TABLE I.

Mean Pressures for Various Methods of Expansion.

Values of $\frac{p_m}{p_1}$. *Adiabatic Expansion of Steam.*

Ratio of Expansion.	Cut-off: $\frac{1}{r}$	Percentage of Steam and Value of n.							
		100 1·135	90 1·125	80 1·115	76 1·111	70 1·105	60 1·095	50 1·085	100 1·333
2	$\frac{1}{2}$	·829	·831	·833	·834	·835	·836	·837	·810
$2\frac{1}{4}$	$\frac{4}{9}$	·785	·787	·788	·789	·790	·791	·793	·754
$2\frac{1}{2}$	$\frac{2}{5}$	·744	·746	·747	·748	·749	·750	·751	·714
$2\frac{3}{4}$	$\frac{4}{11}$	·707	·708	·710	·711	·712	·713	·714	·675
3	$\frac{1}{3}$	·675	·676	·677	·678	·679	·681	·683	·639
$3\frac{1}{4}$	$\frac{4}{13}$	·644	·645	·647	·648	·649	·650	·652	·606
$3\frac{1}{2}$	$\frac{3}{10}$	·633	·635	·636	·637	·639	·641	·643	·600
$3\frac{1}{2}$	$\frac{2}{7}$	·616	·618	·619	·620	·622	·624	·626	·576
$3\frac{3}{4}$	$\frac{4}{15}$	·591	·592	·593	·594	·595	·596	·598	·552
4	$\frac{1}{4}$	·567	·568	·570	·572	·573	·574	·576	·523
$4\frac{1}{2}$	$\frac{2}{9}$	·525	·527	·528	·530	·531	·533	·534	·486
5	$\frac{1}{5}$	·488	·491	·493	·494	·496	·498	·500	·447
$5\frac{1}{2}$	$\frac{1}{11}$	·458	·460	·462	·463	·465	·467	·470	·417
6	$\frac{1}{6}$	·432	·434	·435	·437	·439	·441	·443	·390
$6\frac{1}{2}$	$\frac{2}{13}$	·409	·410	·411	·413	·415	·417	·420	·369
7	$\frac{1}{7}$	·387	·390	·392	·394	·400	·403	·405	·345
8	$\frac{1}{8}$	·355	·356	·357	·358	·360	·361	·363	·312
10	$\frac{1}{10}$	·298	·300	·302	·303	·304	·305	·308	·263
20	$\frac{1}{20}$	·170	·173	·175	·177	·178	·180	·182	·144
50	$\frac{1}{50}$	·080	·082	·083	·084	·084	·085	·086	·063
100	$\frac{1}{100}$	·044	·045	·045	·046	·046	·047	·048	·034

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TABLE II.

Mean Pressures for Various Methods of Expansion.

Values of $\frac{p_m}{p_1}$ *for Steam, Air, Gas and Mixtures.*

Ratio of Expansion, r.	Point of cut-off, $\frac{1}{r}$	Steam Expanding, Dry and Saturated, n, 1·046.	Moist Air in Compressors, n, 1·20.	Steam and Leakage, Actual Engines.		Gas and Vapor in Gas Engine, n, 1·60.	Gases.	
				n, 0·50	n, 0·75		Isothermal, n, 1·00	Adiabatic, n, 1·41
2	$\frac{1}{2}$	·841	·825	·914	·875	·783	·846	·801
2¼	$\frac{2}{9}$	·793	·787	·888	·844	·733	·804	·753
2½	$\frac{2}{5}$	·760	·745	·866	·800	·683	·765	·707
2¾	$\frac{4}{11}$	·717	·700	·846	·785	·638	·731	·668
3	$\frac{1}{3}$	·695	·665	·824	·752	·598	·699	·638
3¼	$\frac{4}{13}$	·665	·635	·802	·732	·578	·670	·596
3½	$\frac{1}{3}$	·652	·625	·796	·716	·568	·661	·588
3½	$\frac{2}{7}$	·632	·605	·782	·704	·548	.642	·568
3¾	$\frac{4}{15}$	·608	·580	·775	·684	·515	·616	·538
4	$\frac{1}{4}$	·587	·550	·750	·664	·486	·566	·518
4½	$\frac{2}{9}$	·540	·510	·720	·624	·441	·555	·473
5	$\frac{1}{5}$	·510	·482	·695	·600	·406	·522	·428
5½	$\frac{2}{11}$	·478	·455	·674	·560	·371	·492	·406
6	$\frac{1}{6}$	·454	·420	·650	·530	·349	·465	·378
6½	$\frac{2}{13}$	·430	·390	·632	·515	·326	·441	·358
7	$\frac{1}{7}$	·409	·375	·612	·500	·303	·421	·337
8	$\frac{1}{8}$	·372	·340	·697	·468	·276	·385	·302
10	$\frac{1}{10}$	·326	·284	·532	·412	·225	·330	·253
20	$\frac{1}{20}$	·192	·165	·396	·272	·103	·200	·138
50	$\frac{1}{50}$	·091	·074	·245	·193	·050	·098	·060
100	$\frac{1}{100}$	·053	·040	·180	‘134	·025	·056	·032

WHAT IS THE MOST ECONOMICAL POINT OF CUT-OFF FOR STEAM ENGINES, CONSIDERED AS A QUESTION OF FINANCE?

By William Dennis Marks,
Whitney Professor of Dynamical Engineering, University of Pennsylvania.

In the Journal of the Franklin Institute for June, 1880, the writer published a brief paper, determining, in an approximate way, from a purely dynamical point of view, the most economical point of cut-off for steam engines. The limited scope of this inquiry, which confined itself entirely to the ratio of the indicated horse-power to the steam used, provoked an amount of criticism in German, English and American serials which was surprising, and the misapprehension of the limitations of the paper, together with the way in which the intent of the writer was gratuitously assumed, finally caused a reply to Professor Thurston in the *Scientific American* for December 4th, 1880, whose severity has been much regretted by the writer, because of the many and great services which Professor Thurston has rendered in the advancement of rational engineering knowledge, and because the matter arose entirely from a misunderstanding of the writer's intentions on the part of the gentleman named.

As a question of finance the subject becomes more complicated, for the engine owner asks not, How can I save the most steam? but, All the circumstances being taken into consideration, how can I get the useful work which I require most cheaply?

So far as the delivery of useful work by the engine alone is concerned, a method has been given by Professor Rankine, and elaborated by himself as well as others; but, perhaps because of insufficiently profound study of the question from a financial point of view, or because the subject was hardly deemed worthy of his thought, the question was never exhausted by him or his followers.

The shipowner says, How can I obtain the power to drive the propeller most cheaply?

The mill owner says, How can I obtain the power to drive the mill stones most cheaply? and the shop owner says, How can I obtain the power to drive my machinery most cheaply? and as they use the

engine for the purpose of making money, wish to have it designed for that purpose, and care nothing at all for purely scientific considerations.

It is to these men that the writer will endeavor to make reply, giving to a most perplexing question, involving many considerations, at least an approximately correct reply, and indicating a method which, by elaboration and a detailed consideration of the thermodynamic questions raised, will, we trust, enable an engineer to reach an economy of useful power, as yet not *knowingly* obtained by other means than an inspection of the profit and loss account at the end of the year.

In "The Limitations of the Steam Engine," JOUR. FRANK INST., August, 1880, the writer stated as the five points of an engine: (1) concentration of power, (2) economy of steam, (3) regularity of speed, (4) simplicity of design and (5) durability of construction.

In that paper and in his work "The Relative Proportions of the Steam Engine" almost every detail of a steam engine, affecting these points has been discussed at length.

When we do not restrict ourselves to economy of nature's forces economy of steam becomes economy of money.

The following assumptions are made and particulars must be understood in the discussion which follows.

The expansion curve of steam is assumed to be, with sufficient practical accuracy, an equilateral hyperbola.

The steam made by the boiler is to the steam shown by the indicator diagram as 4 to 3. This certainly is not correct under all circumstances, but is an approximation derived from the experiments under favorable conditions upon the Reynolds-Corliss, the Harris-Corliss and the Wheelock Engines at the Millers' Exhibition, Cincinnati, June, 1880 (Report of J. T. Hill, pages 77 and 79), and is nearly the same for both condensing and non-condensing engines. Those in possession of more accurate experimental data can substitute other ratios in each case.

A percentage of the true stroke must be added at each end of the sketch which is made to allow for the clearance which must be determined. The cost of all charges upon the engine and machinery is taken in steam for the sake of convenience, and this proceeding is perfectly proper, since money and steam are convertible.

For the present all reference to the saving of fuel resulting from the diminished number of heat units required to increase the pressure

of steam is *premeditatedly* omitted because we are practically limited by expense to low pressures in ordinary cases.

The constant charges which come upon engine boilers and machinery are as follows:

(1) Wages of attendants upon engine, boilers and machinery.
(2) Interest upon cost of engine, boilers and machinery.
(3) Depreciation of engine, boilers and machinery.
(4) Repairs to engine, boilers and machinery.
(5) Cost of lubrication of engine, boilers and machinery.
(6) Taxes and insurance upon engine, boilers and machinery.
(7) Interest upon cost of shelter and room for engine, boilers and machinery.

Many other charges may exist which the writer has not mentioned and some of the charges mentioned are not applicable is cases where methods of charging cost may differ from that of works and factories engaged in the production of some staple article or of a ship where the engine serves only for propulsion. In every case the distribution of cost must form an individual problem.

The only variable charge when the engine is already established is the cost of the steam, *i. e.*, fuel and water. When engaged in the design of an engine and plant most of the constant charges may be regarded as variables, but not according to any uniform law, and must be considered as separate problems which must be solved from known precedents which vary in different localities.

Let P = the mean effective steam pressure in pounds per square inch.

Let C = the mean effective steam pressure in pounds per square inch required to drive the engine and machinery against frictional resistances only.

Let P_b = the absolute initial pressure in the cylinder in pounds per square inch.

Let B = the absolute back pressure in the cylinder in pounds per square inch while the exhaust port is open.

Let e = the fraction of the stroke at which steam is cut off.

Let V = the volume of the steam cylinder.

Let c = the fraction of the volume of steam (of the stroke also) equaling the ratio of the constant charges to the total cost of a cylinder full of steam for any assumed time.

Let b = the fraction of the stroke at which compression begins, being measured from the opposite end from which e is measured.

Let k = the fraction of the stroke allowed for clearance.

We can then write

$$\frac{\text{Useful work}}{\text{Cost of work in steam}} = \frac{(P-C)V}{\left[\frac{4}{3}e + c - b\frac{B}{P_b}\right]V} = \frac{P-C}{\frac{4}{3}e + c - b\frac{B}{P_b}} \quad (1)$$

$$\text{But } P = eP_b\left(1+\text{nat. log.}\frac{1}{e}\right) - b\left[1 - B\left(1-\text{nat. log.}\frac{b}{k}\right)\right] \quad (2)$$

This value of P as well as that of C could be much more accurately determined by the careful use of an indicator.

Substituting in equation (1), we have

$$\frac{\text{Useful work}}{\text{Cost of work}} = \frac{eP_b\left(1+\text{nat. log.}\frac{1}{e}\right)}{\frac{4}{3}e + c - b\frac{B}{P_b}} - \frac{B\left[1-b\left(1-\text{nat. log.}\frac{b}{k}\right)\right] + C}{\frac{4}{3}e + c - b\frac{B}{P_b}} \quad (3)$$

Differentiating with respect to e and seeking a maximum, we have

$$e = \frac{B\left[1-b\left(1-\text{nat. log.}\frac{b}{k}\right)\right] + C}{P_b} + \frac{3}{4}\left(e - \frac{Bb}{P_b}\right)\text{nat. log.}\frac{1}{e} \quad (4)$$

The natural logarithm can be obtained by multiplying the common logarithm by 2·3026, and this transcendental equation must be solved by a series of approximations, beginning with an assumption that

$$\text{nat. log.}\frac{1}{e} = \text{nat. log.}\frac{P_b}{B\left[1-b\left(1-\text{nat. log.}\frac{b}{k}\right)\right] + C}$$

and substituting the nearer value of $\frac{1}{e}$ again in the second member of equation (4), and so on, until two successive values of e nearly agree.

As logarithms do not vary rapidly, the approximations required to obtain all the accuracy justified by the data or realizable in practice will be few.

Perhaps how to deduce the value of c is not clear.

Determine the constant charges, in dollars and cents, upon engine and machinery for one day. Regardless of the power, determine approximately, from the first term only of the second member of equation (4), the most economical point of cut-off for steam alone.

With the following formula, determine the weight of water required per horse-power per hour:

W = weight of water per H.-P., per hour, for no cut-off at all.

S = specific volume of steam for pressure P_b.

$$W = \frac{4}{3} \frac{859380\, e \times \frac{1}{e}}{S \left\{ e P_b \left[1 + \text{nat. log.} \frac{1}{e} \right] - B \left[1 - b \left(1 - \text{nat. log.} \frac{b}{k} \right) \right] \right\}} \quad (5)$$

(See the Limitations of the Steam Engine, Jour. Frank. Inst., Aug., 1880.)

With this determine the cost of fuel and water for full stroke for the required horse-power per day—remembering that $\frac{4}{3}$ is an assumed quantity which, at the best, is only approximately correct.

We will then have

$$c = \frac{\$\text{constant charges per day}}{\$\text{constant charges} + \$\text{cost of fuel and water per day}}$$

c can be redetermined more accurately after e is more accurately determined by its use.

An inspection of equation (4) reveals many interesting facts.

We observe that an increase of the initial pressure or a diminution of the back pressure renders the cut-off of steam earlier. Also that a diminution of the back pressure has much more influence than an increase of the initial pressure. We further observe that any diminution of the frictional resistance of the engine and machinery has a great diminishing effect on the point of cut-off and that the constant charges will largely add to the distance of the point of cut-off from the beginning of the stroke, their influence being particularly felt whenever B and C are small.

Economy of money will give increased concentration of power and simplicity of design.

Regularity of speed, under all circumstances, demands an automatic cut-off, as it is the only effective expedient we now know for that purpose.

The condition is that the most economical point of cut-off shall be determined, and with that an engine designed for the required power.

A lessened demand for power will result in an earlier cut-off, and although the steam is not used quite so economically per horse-power, less steam is used, and the result is a saving of coal.

With an engine already erected, the only means of attaining the most economical point of cut-off is to change the load, the back pressure or the initial pressure.

The quantity C shows the great importance of reducing the frictional resistance in the transmission and accomplishment of work.

To determine the most economical point of cut-off for an established engine, the following method can be used:

(1) Determine the mean absolute back pressure.

(2) Determine the mean effective pressure required to drive engine and machinery against friction at the required speed.

(3) Add (1) and (2) together, and divide by the absolute initial pressure, the result will be the approximate cut-off for the greatest economy of steam.

(4) Determine the constant charges on engine and plant for some assumed time.

(5) Determine the cost of fuel and water required with the given pressure and approximate cut-off by dividing $\frac{4}{3}$ of 859380 by the mean effective steam pressure when doing useful work, multiplied by the specific volume of steam at the initial pressure, and further multiplying this result by the number of hours in the assumed time and the horse-power required. Add the cost of fuel and water to the constant charges, and divide the constant charges by the sum, the result will be the ratio of the constant charges to the total cost of steam and transmission of power.

(6) From this ratio subtract the product of the absolute back pressure during an open exhaust by the fraction of the stroke for compression, divided by the absolute initial pressure, and multiply the result by $\frac{3}{4}$ of the natural logarithm of $\frac{1}{e}$, e being taken at the value given by paragraph (3).

(7) Add the results of (3) and (6) together to obtain a first approximation to the cut-off required.

(8) Find the natural logarithm of the new value of $\frac{1}{e}$, and with

this proceed as in paragraphs (6) and (7) until the successive values nearly coincide.

(9) Should the final value of e differ greatly from that assumed from paragraph (3), a redetermination of the ratio of the constant charges to the total cost of steam and transmission of power (see paragraph 5) may be necessary, and another series of approximations to the value of e will also become necessary.

In closing, the writer wishes to say that he does not feel that this topic is by any means exhausted, and that he trusts that some one will endeavor to give the correct solution of the problem a simpler form.

Philadelphia, Feb. 3, 1882.

CONTRIBUTION TO THE HISTORY OF THE LINK MOTION.

By JOHN L. WHETSTONE, Esq., Cincinnati, Ohio.

To the Committee of Publication of the Franklin Institute.

GENTLEMEN:—In view of the recent proposed changes in the system of link motion for steam engines it seems very desirable that the work already done in the same direction should be recorded. I therefore wrote to my friend Mr. Whetstone for his recollection of a system originated by him, whereby he obtained with one eccentric a reversing link, and employed the motion of the cross-head to give the lap and lead. Also for his method of plotting the ordinary link motion. The paper now submitted has been sent to me for such use as I may desire to make of it. During the time that he was engaged on the work he describes I had full knowledge of his methods, as I was with him in the same establishment, being foreman of the locomotive shop in which he had charge of the drawing room. Mr. Whetstone was one of the most brilliant of the mechanical engineers who at that time turned their attention to the locomotive, but the necessity arising for him to take charge of important interests not involving so directly an attention to this branch of mechanical engineering, his attention has in some measure been directed from it. Submitting his paper with this note as explanatory, I am,

Yours truly, COLEMAN SELLERS.

In the years 1851-2 the writer assisted in the designing and con-

structing some locomotive engines for a special purpose, and the arrangement of the valve gear was especially assigned to him. From the peculiarity of the general arrangement of the machinery it was found impracticable to use more than one eccentric for operating the valves of each engine, and it was necessary to use a valve with considerable lap. There being barely room for one eccentric for each engine on the driving axle, the device of shifting a lead eccentric across the axle for the purpose of obtaining lead for the forward and backward movements could not be applied. The valve gear which was finally adopted was substantially the same as represented in Fig. 7, with slight modifications. The eccentric was set so as to be at half throw when the crank pin was at the ends of the stroke or at the dead point, and connected by a rod to an arm on the rocker shaft having at its other end a double arm, carrying a link bar for the purpose of giving reversing movements to the valve. The position of the rocker arm and link bar was therefore the same when the crank was at either end of the stroke, viz., that shown in the Fig. 7, at *A B C*. For the purpose of giving the advanced movement requisite for the lap and lead of the valve, the shifting or reversing rod (one end of which is properly swiveled to the link bar) is connected to the fulcrum *F* of a lever, the longer arm of which is suitably attached to the cross-head of the engine, the shorter arm being geared to the valve rod at *G*, or to any suitable device necessary to transmit the movement to the valve. The length of the arms of this lead-lever are such that when the fulcrum *F* is at the half throw of the eccentric the upper wrist of the lever *G* is removed from the centre line *H K* to the extent of the lap and lead of the valve, the longer end connected with the crosshead then at one end of the stroke, and if the cross-head be at the other end of the stroke, the upper wrist will be as far removed to the opposite side of the centre line.

It will also be observed that the position of the wrist operating the valve rod will remain the same, in whatever part of the link bar the reversing bar or rod *D F* may be situated, whether in full gear forward or backward, or at any intermediate point, and the lead of the valve will be the same at both ends of the stroke. The throw of the eccentric for this valve gear will be shorter than the travel of the valve, inasmuch as part of the valve movement is obtained from the crosshead. In practice it is found that about two-thirds of the lead of the valve is obtained from the crosshead, and

the eccentricity of the eccentric is lessened to that extent. At first sight it would seem as though the whole was due to the cross-head, but it must be borne in mind that during the last half of each stroke of the piston the eccentric motiou is in a direction opposite to that of the crosshead, thus combining the two movements to extend the travel of the valve. The effect of this combination is to accelerate the movement of the valve at the opening of the ports, and to retard it at and towards the end of the throw of the valve, thus giving a longer admission of steam with a given lap of valve then by the eccentric motion alone. It would be quite practicable to operate the rocker arm link of one engine from the crosshead of the opposite engine, the lead being obtained from its own crosshead, and probably the greatest objection to such an arrangement would arise from the fact that the disability of one engine through an accident would render the valve movement of the other engine inoperative.

In 1853-4 the link motion, as it is termed (combining the movement of the forward and backward eccentrics), began to be generally adopted on locomotives, in place of the hooked or forked eccentric rods previously employed for reversing movements of the engine, in connection with separate eccentrics and valves arranged for the purpose of cutting off the steam at different parts of the stroke. The almost universal employment of the link motion at the present time in engines of every description, which are required to run at high rates of speed, affords evidence of its acknowledged superiority over other forms of valve gear.

At the time of their first introduction, the great difficulty of adjustment of the working parts so as to produce equality of cut-off or suppression of the steam on both strokes of the piston at all desired positions of the link gear operated as a serious objection to their use. The writer being then engaged in designing and constructing locomotives in a large establishment determined, if possible, to devise a system of arrangement of the various parts of a link gear which should secure the above results. The difficulty of this problem is enhanced by reason of the variety of elements modifying the movements of the valve, and which must be taken into the account in its solution—as for instance the proportionate lengths of the main connecting rod and crank, the lap and lead as compared with the travel of the valve, length of rocker arm, radius of link, all of which to some extent require modifications of the link gear. Adopting as a motto the aphor-

ism of Lord Bacon that Nature can only be conquered or rendered subservient to our purposes by implicit obedience or compliance with her laws, the writer determined to find the several positions of the link requisite to produce suppression or cut-off at the principal points in the stroke of the piston, in forward and back gear, and then endeavor to arrange the reversing gear, so as to maintain the link in the position so ascertained. For this purpose the positions of all the main lines and centres of the several parts of the engine involved were accurately laid down of full size on a drafting table, viz., the centre line from cylinder to driving axle, showing centre of crosshead wrist at each end of the stroke, and at half stroke, and three-quarters and seven-eighths stroke each way; they are described by the centre of wrist on rocker arm to be operated by the link on which are marked the lap and lead for the valve, as previously determined upon; the centre line from driving axle bisecting the arc described by the rocker arm in its travel. In the accompanying illustrations, this last named line is coincident with the centre line from cylinder to driving axle. The circles

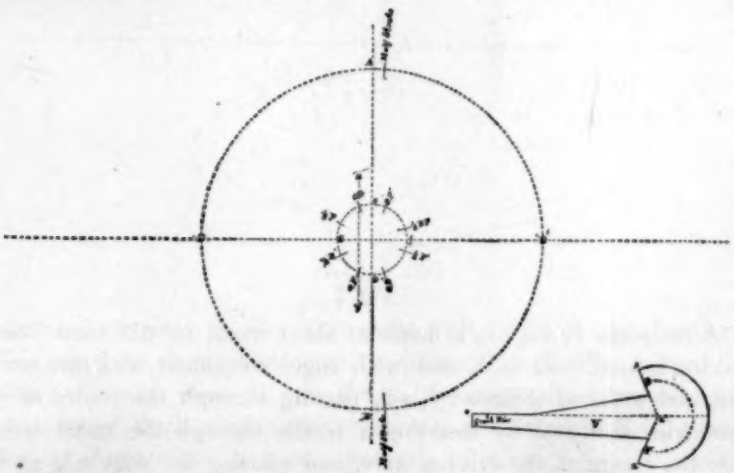

Fig. 1.

O A O' B, Figs. 1, 2, represents the circuits described by the crank wrist, and *D E c f* the circuit described by the centre of the two eccentrics. The positions of the crank pin at the different parts of the stroke are obtained by using a tram or beam compass set to the length of the distance between the centres of the driving axle and that of the crosshead wrist at half stroke shown at ½ on line *G H*, Fig. 3 (thus

representing the length of the main connecting rod), one leg of tram being placed at the desired point in the stroke, with the other leg arcs are described intersecting the crank circle *O A O' B* at points marked half stroke, Fig. 1, and $\frac{1}{8}F$, $\frac{1}{8}B$, $\frac{1}{8}F'$, $\frac{1}{8}B'$, Fig. 2, the position of the forward and back eccentric are determined by drawing a line, *A B*, Fig. 1, through the centre of the driving axle at right angles to the centre line extending to the rocker arm to be operated by the link, and another line, *X Y*, parallel to *A B*, and distant from it equal to the amount of lap and lead to be given to the valve, the points of intersection with the circle *D E c f* will represent the centres of the two eccentrics when the crank pin is at *O* at the end or commencement of the stroke.

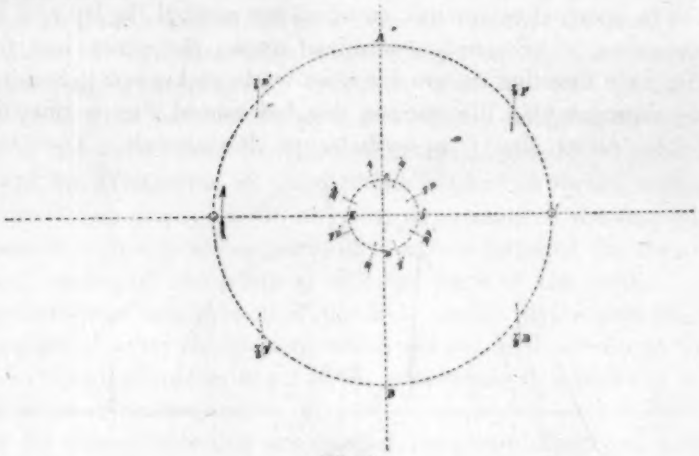

Fig. 2.

A template *V*, Fig. 1, is made of sheet metal or stiff card board, having a small hole at *z*, and with edges coincident with the crank pin, and with radial lines *z* 1, *z* 2, passing through the centre of the eccentrics *F B*, and by inserting a needle through the small hole *z* into the centre of the driving axle, and placing the edge *z c* at the intersections representing the crank pin at various portions of the stroke, the positions of the eccentrics at each position of the crank can be marked with a pencil or pen intersecting the circle *D E c f*. Thus the forward and back eccentrics respectively will be at *O F*, and *O B*, Fig. 1, when the crank is at *O;* at $\frac{1}{2}$ *F*, $\frac{1}{2}$ *B F* when crank is at half stroke; at *O F'* and *O B'* with crank at *O'* and at $\frac{1}{2}$ *F'* and $\frac{1}{2}$ *B F'*, with crank at the other half stroke. By a similar procedure the rela-

tive positions of the eccentrics at any other desired point of the stroke, say seven-eighth stroke, is obtained as shown in Fig. 2, where F and $B\,F$ are the centres of the forward and back eccentrics with crank at $\frac{7}{8}\,F$, moving in the direction of the arrow, and F' and BF', with crank at $\frac{7}{8}\,F'$, and B and $F\,B$, with crank at $\frac{7}{8}\,B$, and B' and $F\,B'$, with crank at $\frac{7}{8}\,B'$, moving in the opposite direction.

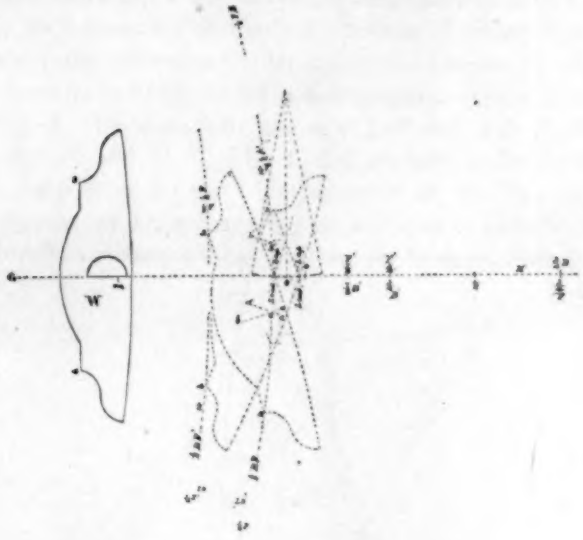

Fig. 3.

Another template W, Fig. 3, is made with a curved edge having a radius equal to the distance from the points C or D, Fig. 1, 2, to the rocker arm wrist when at mid throw, the corners 3 4 representing the centres of the knuckles connecting with the eccentric rods, a hole being cut through the template, one side of which coincides with a straight line dividing the link into two equal portions. The length of the eccentric rods is obtained by placing the curved edge of the link template opposite to the corner 3 in contact with the rocker arm wrist centre when at mid throw, at O', Fig. 3, and moving the body of the template to a position where the same tram will touch the points D, Figs. 1 and 3, on the template, and C, Fig. 1 and 4 on the template. With the length thus obtained as a radius, and the centres at

the intersection $\frac{1}{2}$ *F* and $\frac{1}{2}$ *F'*, Fig. 1, describe the arcs $\frac{1}{2}$ *F*, $\frac{1}{2}$ *F'* above the centre line *G H*, Fig. 3, and from the centres $\frac{1}{2}$ *B F* and $\frac{1}{2}$ *B F'*, Fig. 1, describe below the centre line the arcs $\frac{1}{2}$ *B F* and $\frac{1}{2}$ *B F'*, Fig. 3. The lap of the valve having been marked as shown in Fig. 3, the template *W* is now placed with its corner 3 in contact with the

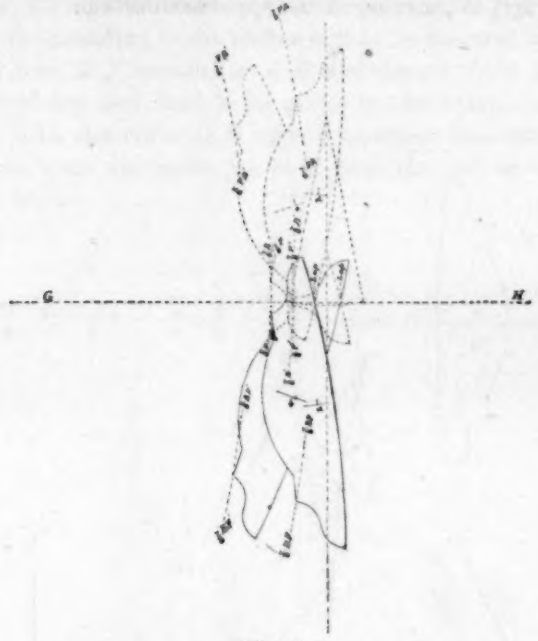

Fig. 4.

arc $\frac{1}{2}$ *F*, and corner 4 in contact with arc $\frac{1}{2}$ *B F*, and moved along until it reaches the place indicated in Fig. 3, where the curved edge cuts the point marked *Lap*, being the point of suppression or cut-off, and with a pen or pencil the line *a b* is drawn along the straight side of the hole in the template. By a similar process, using the arcs $\frac{1}{2}$ *F'* and $\frac{1}{2}$ *B F'* the line *c d* is obtained, and their intersection at *e* determines the vibrating centre of the link and its position to cut-off at half stroke on the forward movement of the engine in the direction of the arrow, Fig. 1. By a similar process the intersections at *g* are obtained, being the position of the link-vibrating centre for the backward movement of the engine. The vibrating centre of the link should be marked on the template at *m*.

The next step to be taken is to obtain the position of the vibrating

centre of the link in order to cut off the steam at seven-eighths stroke of the piston in forward and back gear. For this purpose, with the centre *F*, Fig. 2, and the length of eccentric rod for radius as before, the arcs $\frac{7}{8}$ *F*, Fig. 4, is described downwards from the centre line and a little above it, each end of the arc being suitably marked to identify it, and from the centre *B F*, Fig. 2, the arc $\frac{7}{8}$ *B F*, Fig. 4, is described at some distance below the centre line, and suitably marked. By traversing the corners 3 4 of the link template *W* along these arcs until the curved edge intersects the lap point, the position of the vibrating centre in order to effect cut-off at seven-eighths stroke is found to be at *a*, Fig. 4. By describing the arcs $\frac{7}{8}$ *F'* and $\frac{7}{8}$ *B F'*, Fig. 4, from the centre *F'* and *B F'*, Fig. 2, and traversing the template *W* as before, the position for the vibrating centre of the link for cut-off at seven-eighths stroke in forward gear is found to be at *b*. By similar procedure the corresponding positions for back gear are found at *c d*, Fig. 4.

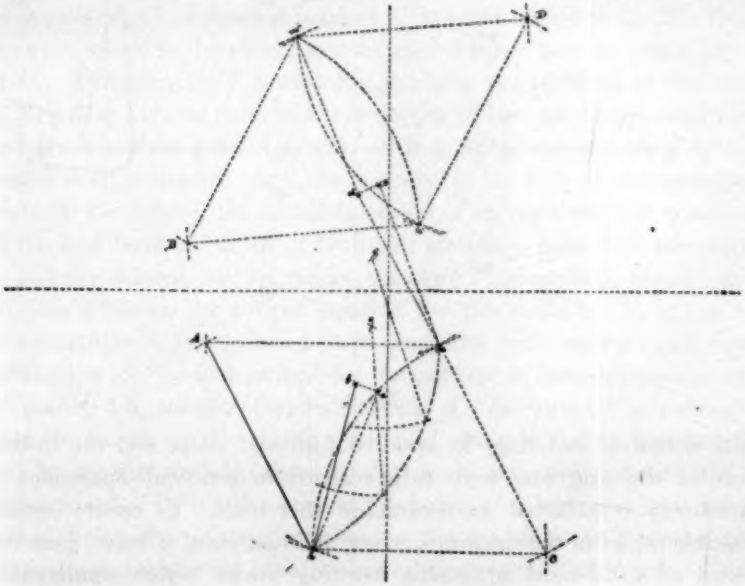

Fig. 5.

Having now ascertained the precise vibrating centre of the link, and its required positions to effect equal suppression of steam at the most important parts of the stroke both in forward and back gear, it

now remains to adjust the reversing gear to meet those requirements. Fig. 5 represents at *a b c d e g* the several points determined in the other drawings. The length of the suspending or sustaining bar on which the link vibrates being determined upon, with this length for radius the intersecting arcs *h* and *h'* are described from *a* and *b* as centres, and *m* and *m'* from the centres *c* and *d*. Then with the centres *h m* with the length of the lifting arm for radius the intersecting arcs *A C* are described, and from *h'* and *m'* the intersecting arcs *B* and *D*, all which points indicate a possible position for the reversing shaft. The reversing shaft may be located at any of the points *A B C* or *D*, though in practice the writer always adopted that shown at *A* in all the locomotives constructed under his direction. In practice all the operations were performed on one drawing by describing the arcs of the eccentric rods in different colored inks, and

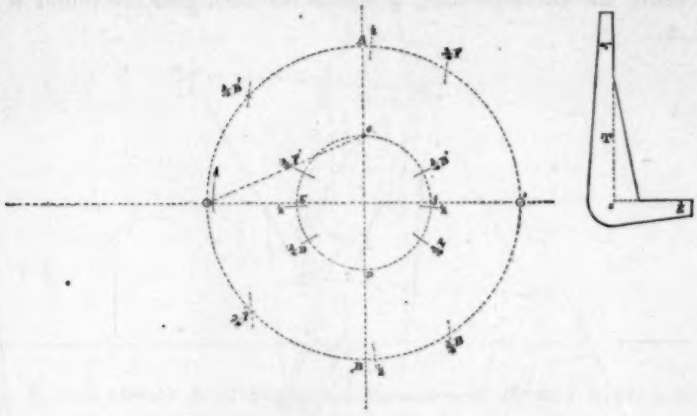

Fig. 6.

with dotted or full lines to avoid confusion of lines, and the indications of the diagrams were fully realized in practical operations of machinery constructed in accordance therewith. To ensure against possible errors in the drawings, every different class of valve gear was tested on a full-sized adjustable working model before commencing on a working machine. With a well-constructed working model capable of adjustment to all the varied positions and proportions for cranks, eccentrics, rocker arms, connecting rods, etc., with links of proper radii, the position for the vibrating centre of the link at the various points of suppression or cut-off can be readily obtained

after careful adjustment of the eccentrics and rods so as to obtain the correct lead to the valve, when the piston is at either end of the stroke, by moving the vibrating centre of the link on the central line as *G m*, Fig. 3, until the position *m* is obtained such that with the piston at either half stroke, when the link is moved to the point of suppression for either stroke, the vibrating centre shall be at the same point, which can be ascertained after a few trials. The exact position on the link for its vibrating centre being determined, if the crosshead is set at three-fourths or seven-eighths stroke for each stroke, the position of the vibrating centre of the link can be noted for the point of suppression at each stroke, both in forward and back gear, and the position for the reversing shaft can be determined as explained for Fig. 5.

Fig. 7 represents the general arrangement of rocker arms, link and crosshead attachment for a valve movement with a single eccentric, or its equivalent, which in this case, Fig. 6, is a wrist on the end of one arm extending from the crank pin toward the axle centre, and adjusted so as to occupy the position marked *C* at right angles to the line from the axle centre to the rocker arm at mid-throw when the crank pin is at *O*. The template *T* is used for marking the positions of the wrist *C*, Fig. 6, at various parts of the stroke, as in the case before described, and the template *a b c d e f g h*, with a hole corresponding to the rocker shaft is used to mark the positions of the link at corresponding parts of the stroke, the dotted lines *k m*, *n m*, represent the positions of the lead lever to cut off at both half strokes. Now with the centre *p*, and the shifting bar for radius, the arc *s t* is described, whose intersections 2 3 show the proper position for the shifting bar to cut off at half stroke in forward and back gear, and with centre *r* and same radius, the arc *u v* is described for the purpose of determining the cut-off points, 4 5, for the other half stroke. By marking off on a straight edge the length of the shifting bar and the point of suppression *D*, and laying this straight edge at *p* 2, *p* 4, the points of suspension *c' d'* are obtained. By similar process the points *a' b' e' f' g' h'* are ascertained, being the points of suspension for cut-off at half stroke and three-quarter stroke for both strokes in forward and back gear. From these points as centres the intersecting arcs *i' k' m' n'* are described, with the suspending bar for radius, and by bisecting the chords *i' m'* and *k' n'*, the centre for the reversing shaft *x* is found, and the length of reversing arm determined. The reversing shaft may be placed above the centre of the rocker shaft, and its position ascertained by

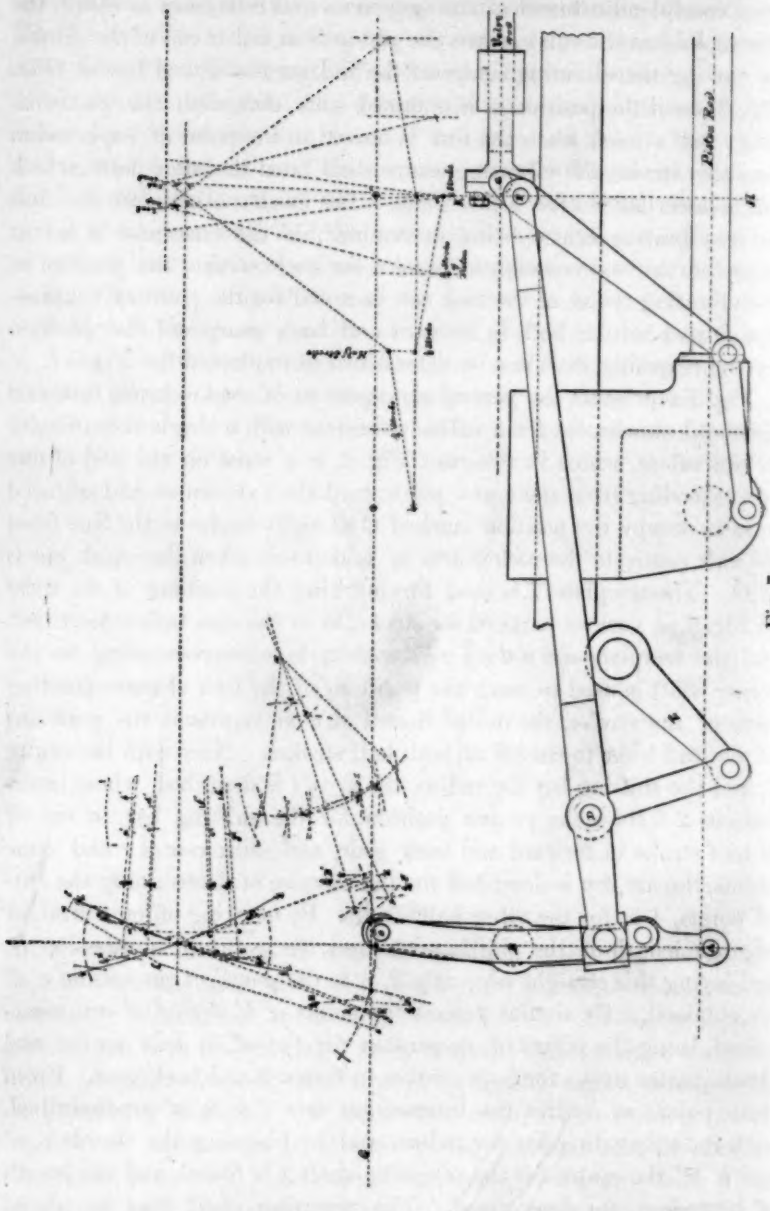

Fig. 7.

similar process; but when practicable the position below the rocker shaft is preferable. This style of valve gear is more especially applicable and adapted to freight engines having small drivers, inasmuch as all the valve gear is readily within sight and reach, and with a given lap and travel of valve a longer admission of steam in full gear is attained, with the same range of cut-off as in the shifting link. The uniformity of lead at all positions in gear is not favorable to the attainment of the highest speeds required in passenger traffic.

A NEW THEORY OF THE SUSPENSION SYSTEM WITH STIFFENING TRUSS.

By A. Jay DuBois, Ph.D.

Professor of Dynamic Engineering in the Sheffield Scientific School of Yale College.

The points in which the present discussion differs from the theory generally received, are as follows:

The ordinary suspension system is assumed to consist of cable, truss and stays. The cable is supposed to carry the entire dead and full live load. The stays are inserted for additional stiffness at the flanks, while the truss is constructed to merely rest upon the abutments, and it is assumed that it so distributes any partial loading that the curve of the cable remains practically unchanged. That is, the curve of the cable is assumed to be parabolic and it is assumed that the truss distributes any partial load *so as to make it take effect upon the cable as a uniform load*, thus preserving the parabolic shape.

In opposition to this generally received theory, we maintain in our present discussion that the curve of the cable does *not* remain parabolic, but takes the curve of equilibrium due to the loading. We thus claim to obtain a more accurate, rational and scientific theory of the stiffening truss. We also discard stays entirely, upon the ground that they are unnecessary. Finally, we suppose the truss firmly bolted down at the ends, so that the tangent to the curve of deflection at the ends is always horizontal, and that it supports its proper portion of the live load.

All of these points are too obvious to need defence. The common assumption that the curve of the cable remains always a parabola, and that the truss distributes the load *uniformly*, every engineer knows to

be false. Such an assumption cannot fail to lead to an incorrect theory, and it is not to be wondered at that trusses proportioned in accordance fail to give satisfactory rigidity. One only needs to watch the oscillation and deflection due to a passing load, to realize how erroneous such an assumption must be, yet it lies at the bottom of every received theory.

As to the stays, we regard their use as a confession of imperfect design. Cable and truss—the one to support and the other to stiffen—are sufficient for a rigid system. The introduction of stays renders the combination indeterminate as to its strains and impossible of accurate adjustment. If the material in the stays of many of our suspension bridges were properly put into the truss there would be a positive gain of rigidity.

As to the truss, while it has been recognized that it is the main element of rigidity, owing to false ideas of economy and a superstitious dread of "temperature strains," the very rigidity which is sought to be secured by its use is often sacrificed by the introduction of hinges and allowing the truss to swing free; or, at most, merely rest upon the masonry. There seems no defence for such practice. As stiffness is what we wish, let us take the best means to secure it. The girder fastened down horizontally at both ends is the stiffest construction we can employ. This need not prevent the use of friction rollers at the ends. As to temperature strains in general, they can be easily calculated, are found not to be excessive and it is far better to allow for them by proper cross-sections than to sacrifice rigidity in order to avoid them and then insert all the material, apparently saved, in the shape of stays.

We have allowed ourselves the following preliminary remarks in order to emphasize the points of difference of the present discussion and to point out wherein it is new. The formulæ here given are, so far as we know, entirely new, and the method of calculation we believe to be more exact than any hitherto proposed.

Composite System.

A very common construction for long spans is that shown in Fig. 1. Such a structure we may call "composite"—that is, it consists of two different systems which act together. Fig. 1 represents the most important of these, known as the "suspension system." It consists of a flexible chain or cable which is stiffened under the action of partial

loads by a truss. The truss is slung on to the cable by suspenders and may be of any design, either double or single intersection, Post, Pratt, etc. The cable carries the entire dead weight of the truss—that is, the suspenders are screwed up until the ends of the truss just bear on the abutments. The office of the truss is then mainly to stiffen the cable and prevent change of shape and oscillations, due to partial and moving loads. It also acts to support a share of the moving load. There are usually side spans at each end. In any case the cable is not attached to the towers but passes over rollers at the top and is carried beyond and firmly fastened to large anchorages of masonry.

Defects of the System.

The principal defect of this system is its lack of rigidity. The cable possesses no inherent rigidity except such as is due to its weight and inertia, and any stiffness which it may have, therefore, is due, almost entirely, to the truss.

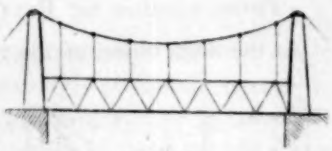

Fig. 1.

A second disadvantage is that a rise of temperature, by decreasing the deflection of the cable, throws considerable load upon the truss. To obviate this objection, the truss is often hinged at the centre and placed on rollers at the ends.

Advantages of the System.

It is evident from the preceding that the system is most advantageous for very long spans. The cable then carries the dead weight in the most advantageous manner and, by reason of its own very considerable weight in such case, resists in some degree the deforming action of partial loads. The truss is then very light compared to what it would have to be if there were no cable.

Stays Unnecessary.

The system is accordingly, in practice, applied only to very long spans. But owing to a lack of rigidity even in such cases, additional stiffness is sought to be obtained by the introduction of *stays*, reaching from the top of the tower to various points of the truss, as shown in Fig. 2. The use of these is not to be recommended, for two reasons. They render the correct determination of the strains indetermi-

nate. A load at any point may be carried entirely by the suspender and stay at that point, or by the suspender and truss, or by the stay and truss. It is impossible to tell exactly the duty performed by each, and even if it were, it would be impossible to so adjust the several systems that they shall take their proper share. If such adjustment were possible it would not last. Variations of strain, set and elongation of the metal, shocks and vibrations, and rise and fall of temperature, would constantly disturb the adjustment.

In the second place, the stays are unnecessary. The truss is a rigid construction in and of itself, and ought to render rigid any system of which it is a member. If properly proportioned, then, the truss is sufficient. Stays are superfluous additions and their use is unscientific. Their employment is a confession of improper design in the truss. We shall, therefore, suppose in what follows that no stays are employd.

Truss should be Fixed Horizontally at the Ends.

As the main object of the truss is to secure rigidity, that construction of truss should be adopted which promises the greatest stiffness. The truss should, therefore, be securely and firmly bolted down at several successive points at the piers and abutments, so that, under all circumstances, it is *fixed horizontally at the ends*. It should also be *continuous and without hinge at the centre*.

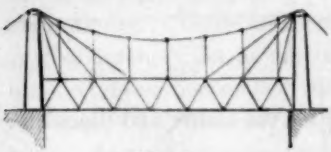

Fig. 2.

It may be objected to this, that in such case there are strains due to change of temperature. This is quite true. But such strains can be accurately found and the truss proportioned to withstand them.

The common practice of hinging the truss in the centre and placing it on rollers at the ends, in order to avoid temperature strains, is at the expense of rigidity. Moreover, nothing is gained in economy when the endeavor is made to make good this loss of rigidity by the employment of stays. The material saved in the truss and cable is balanced by the material in the stays, and the result is a shaky and unscientific combination. It is far better to put the material of the stays into the truss, where it is needed to resist temperature strains, and do away with all hinges and rollers. We thus obtain the greatest stiffness the system admits of, and have a combination the strains in

which can be accurately determined, easy of adjustment and which once adjusted will remain so.

Instead of rollers at the ends, a *sliding joint* at the centre of the truss may be permitted, since such a joint, if properly constructed, does not break the continuity of the flanges, and hence does not impair the rigidity, while it does reduce the strains due to temperature. Rollers may also be used, provided that they do not affect the condition that the truss shall be horizontally fixed at the ends.

Best Form of Suspension System.

The best form of the suspension system then, and the one which we shall investigate in the following pages, consists simply of truss, cable and suspenders, as shown in Fig. 1. The truss is assumed to be fixed horizontally at the ends and to have, if desirable, rollers or a sliding joint at the centre or ends, which does not interfere, however, with the continuity of the flanges at that point.

The truss may be of any of the usual patterns. The flanges are usually horizontal and the bracing either single or double intersection with vertical posts, or triangular. This, however, is by no means necessary. Any form of truss may be employed, whether the flanges are horizontal or parallel, or not.

It is necessary to point out that such a system as the above does not at present exist. Of the large suspension bridges in existence, none are fixed horizontally at the ends, and most employ stays and are hinged at the centre. Our discussion and formulæ, therefore, *do not apply to such*, and will not enable one to find the strains in them. Indeed, from the preceding, we see that it is not possible to find the strains in them with any degree of certainty, and hence it is useless to devote space here to their discussion. It is not, therefore, surprising that the system has been considered, and is still considered, unsatisfactory in railroad practice, except under certain regulations as to allowable speed and load and, as regards its calculation, still more unsatisfactory in theory.

We claim that the system proposed secures the greatest rigidity possible to the combination and admits of satisfactory calculation, and is, therefore, not only the best but the only scientific combination. Finally, that the bugbear of temperature strains is one only in appearance, since what is saved in the truss and cable, by the ordinary system, is lost in the stays.

Theory of Composite Structures Generally.

Before proceeding to investigate the suspension system, we shall illustrate the principles which must govern the discussion of any composite system.

Let λ_1 be the alteration in length, per unit of length, due to the strains in one system, and λ_2 that due to the strains in the other. Then, in order that there may be the same unit strain at all points in both members, we must have

$$\lambda_1 = \lambda_2 \qquad (I)$$

But a common unit strain in both systems implies that both act together and are made of the same material. Equation (I), then, expresses the condition that both systems reach the elastic limit simultaneously, *provided they are both made of the same material.*

In any special case, as we shall see hereafter, λ_1 and λ_2 can easily be found in terms of the depth, span and deflection. Since the span and deflection are the same for both systems, they will cancel in equation (I) and we shall then have from it the relation that should exist between the depths, for the least amount of material. Equation (I), then, under the limitation of a common material, enables us to determine the dimensions of each system, in order that the elastic limit may be simultaneously reached by both—that is, in order that the two may act together as one system.

But whether both systems are made of the same material or not, or whether the elastic limit is reached simultaneously or not, still, if both systems are rigidly connected, they must have a common deflection at each point of connection. One system cannot deflect without causing the other to deflect an equal amount. If then δ_1 is the deflection of one system and δ_2 that of the other, we have

$$\delta_1 = \delta_2 \qquad (II)$$

Since the deflection varies directly as the load, the ratio $\frac{\delta_1}{\delta_2}$ found from equation (II) in any special case, will give the ratio of the loads. If then we know the total load, we can find the load carried by each system.

The two principles expressed by equations (I) and (II) lie at the foundation of the discussion of all composite structures.

Change of Length.

The coefficient of elasticity, E, is that theoretical unit force which would stretch a piece an amount equal to its original length, if the law of proportionality of elongation to stretching force were to hold good throughout such extreme elongation.

Thus if S is the stretching force and F the cross-section, $\frac{S}{F}$ is the unit force. If the elongation produced by this unit force is λ per unit of length, and the length of the piece is L, the elongation due to the unit force $\frac{S}{F}$ is λL. It will then, according to the assumed law, require as many times $\frac{S}{F}$ to produce the elongation L, as λL is contained in L.

Hence,
$$E = \frac{S}{F} \times \frac{L}{\lambda L} = \frac{S}{F\lambda}.$$

From this we have for the elongation per unit of length

$$\lambda = \frac{S}{FE} \qquad \text{(III)}$$

From this equation we can always find the elongation per unit of length, λ, when the cross-section of the piece, F, and the applied force, S, are known.

Application of Preceding Principles.—Illustration.

Let us take as an illustration of these principles and of the use of our fundamental equations (I) and (II) a simple composite structure, represented in Fig. 3.

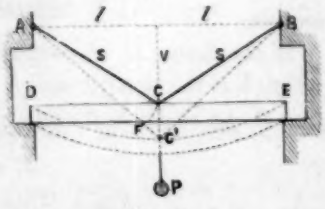

Fig. 3.

It consists of a beam, DE, and two tie rods or stays, AC and BC, united to the beam at its centre, C. A single weight, P, is placed at the centre.

Let Δ_1 be the deflection of the stays, C, C', and Δ_2 the deflection of the beam. Let λ_1 be the coefficient of elongation for the stays and λ_2 for the beam. Let the length of each of each stay be s, the half-span be l and the vertical projection of each stay be v.

1. Deflection of Stays.—The increase of length of each stay is λs. Draw CF at right angles to AC'. Since $\lambda_1 s$ is very small, $AF = AC$ approximately, and hence $FC' = \lambda_1 s$. We have then by similar triangles

$$\frac{CC'}{FC'} = \frac{s}{v} \text{ or } \frac{\Delta_1}{\lambda_1 s} = \frac{s}{v}$$

From this we obtain

$$\Delta_1 = \lambda_1 \frac{s^2}{v} \tag{1}$$

Let nP be the portion of the weight carried by the stays. Then $(1-n)P$ is the portion carried by the beam.

Each stay then carries $\frac{nP}{2}$. The strain in each stay is then

$$\frac{nP}{2} \text{ sec. } \theta = \frac{nP}{2} \frac{s}{v}.$$

From equation (III), the elongation caused by the strain is

$$\lambda = \frac{S}{F_1 E_1} = \frac{nPs}{2F_1 E_1 v}$$

Inserting this value in (1) we have

$$\Delta_1 = \frac{nPs^3}{2F_1 E_1 v^2} \tag{2}$$

where $\frac{nP}{2}$ is the weight carried by the stay and E_1 F_1 are the coefficients of elasticity and the area of cross-section of stay.

2. Deflection of Beam.—The deflection of a beam, supported at the ends, with a weight, $(1-n)P$, in the middle, is from the theory of flexure $\Delta_2 = \frac{(1-n)Pl^3}{6E_2 I_2}$ where I_2 is the moment of inertia of the cross-section.

If the beam is of rectangular constant cross-section, $I_2 = \frac{1}{12}bh^3$, where b is the breadth and h the height of cross-section. The deflection at the centre is therefore

$$\Delta_2 = \frac{2(1-n)Pl^3}{E_2 F_2 h^2} \tag{3}$$

where E_2 is the coefficient of elasticity, F_2 the area of cross-section.

Let ρ, Fig. 4, be the radius of curvature of the neutral axis of the beam. Considering the curve of deflection as an arc of a circle, which is approximately true when the unit strain is constant, or when the deflection is small, as is the case in practice, we have $\Delta_2 : l :: l : 2\rho - \Delta_2$, or neglecting Δ_2 as very small in comparison with 2ρ, $\Delta_2 = \frac{l^2}{2\rho}$.

But when Δ_2 is very small, the length of arc is the same as the

length of chord, approximately. The length of the neutral axis, then, is $2l$ and of the lower edge $2l+\lambda_2 2l = 2l\,(1+\lambda_2)$. Since the lengths are proportional to the radii, $\frac{\rho}{\rho+\frac{h}{2}} = \frac{2l}{2l(1+\lambda_2)}$, or $\rho = \frac{h}{2\lambda_2}$.

Substituting this value of ρ in the value for Δ_2 above, we have

$$\Delta_2 = \frac{\lambda_2 l^2}{h} \tag{4}$$

We have thus found in equations (1) and (4) the deflections in terms of the elongation, and in equations (2) and (3) the deflection in terms of the load.

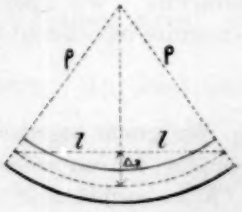

Fig. 4.

We are now ready to apply our principles.

1. To find the best ratio of $\frac{v}{l}$. Since the deflections at *C*, Fig. 3, must be equal, we can equate equations (1) and (4). We thus have

$$\Delta_1 = \Delta_2 \text{ or } \lambda_1 \frac{s^2}{v} = \lambda_2 \frac{l^2}{h} \text{ or } \frac{v}{h} = \frac{\lambda_1}{\lambda_2}, \frac{s^2}{l^2}. \tag{5}$$

Equation (5) gives then the best ratio of $\frac{v}{l}$, whether the material is the same in both systems or not, if we put for λ_1 and λ_2 their values at the limit of elasticity for each material.

If each system is made of the same material, $\lambda_1 = \lambda_2$ and

$$\frac{v}{h} = \frac{s^2}{l^2} \tag{6}$$

2. To find the load carried by each system.—If we equate (2) and (3) instead of (1) and (4), we have

$$\frac{nPs^3}{2F_1E_1v^2} = \frac{2(1-n)Pl^3}{F_2E_2h^2} \tag{7}$$

From this equation we can easily find n, whether the material in the systems is the same or not.

If, however, the material is the same, we have from (4) and (3) $\lambda_2 = \frac{2(1-n)Pl}{E_2F_2h}$, and if we put this equal to the value for λ_1, we

have $E_2 = E_1, \frac{2(1-n)Pl}{F_2 h} = \frac{nPs}{2F_1 v}$, or if we introduce the best ratio of $\frac{v}{h}$ as given by (6)

$$\frac{2(1-n)Pl}{F_2 h} = \frac{nPl^2}{2F_1 sh}, \text{ or } n = \frac{4F_1 s}{F_2 l + 4F_1 s} \tag{8}$$

The preceding is sufficient to illustrate the use of our equations (I), (II), (III) and the method which must be adopted for any composite structure. We shall now proceed to discuss in a similar manner the structure represented by Fig. 1.

Notation.

We group together here the principal notation, which we shall employ, for convenience of reference.

E_1 = coefficient of elasticity of the cable.
E_2 = coefficient of elasticity of the truss.
F_1 = cross-section due to strain at centre of cable.
F_2 = cross-section due to strain at centre of truss.
F_0 = cross-section due to strain at ends of cable.
λ_1 = elongation per unit of length of cable at centre.
λ_2 = elongation per unit of length of truss at centre.
λ = elongation per unit of length of cable at any point.
δ_1 = deflection of cable at any point.
δ_2 = deflection of truss at any point.
Δ_1 = deflection of cable at centre.
Δ_2 = deflection of truss at centre.
I_1 = moment of inertia of cable, I_2 of truss, both at centre.
f_1 = greatest allowable unit strain for cable, f_2 for truss.
s = length of cable supporting dead load only.
t = number of degrees rise or fall above or below mean temperature.
ε = elongation per unit of length due to a rise of temperature of one degree.
q = unit load due to rise or fall of temperature.
h = depth. $2l$ = span. v = versine of cable.
r = height of towers. p = permanent or dead unit load.
m = moving or live unit load.
M_x = moment at any point.

S_x = shear at any point.
M_1 = moment at left end of truss.
M_2 = moment at right end of truss.
H = horizontal pull of cable.
V_1 = vertical reaction of cable at left.

Equilibrium Curve.—Horizontal Force Constant.

The curve or polygon which a perfectly flexible string, hung from two fixed points, assumes when acted upon by a given distributed load or by given concentrated loads, is called the "equilibrium curve" or polygon.

In Fig. 5, let A and B be the ends of the string. In order that these ends may be fixed, we must have at A and B the upward forces V_1 and V_2. The sum of these upward forces must equal the sum of the downward forces. Also if A and B are on the same level, the moment of V_1 with reference to B must be equal and opposite to the sum of the moments of the downward forces with reference to B. So also for V_2 with reference to A. If the forces P_1, P_2, etc., are known and their points of application given, we can then easily find V_1 and V_2. But in order that A may be fixed, we must have not only the upward force, V_1, which can be found as above, but also a horizontal force, H, equal and opposite to the horizontal pull of the string at that point.

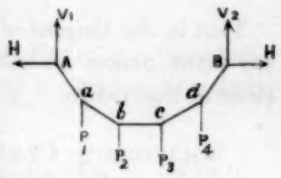

Fig. 5.

The portion of the string Aa must then lie in the direction of the resultant of H and V_1, and the tension in Aa must be equal to that resultant.

The portion ab must then lie in the direction of the resultant of Aa and P_1 and the tension in it is equal to that resultant. But the resultant of P_1 and Aa is the same as the resultant of H, V_1 and P_1. In like manner the tension in any portion, as bc, is the resultant of all the forces to the left or right of that portion.

But all these forces are parallel and vertical, except H, which is horizontal. The tension of any portion, as bc, then, is the resultant of $V_1 - P_1 - P_2$ and H.

But $V_1 - P_1 - P_2$ is the shear just to the right of b. Therefore, at any point of an equilibrium curve or polygon, *the vertical load is the*

shear at that point and the horizontal force at every point is constant and equal to the horizontal pull at A or B.

As the loads are more numerous, the polygon approaches a curve. When the load is distributed, we have an equilibrium curve, Fig. 6. The tension in any element of this curve at any point, as ds, is then the resultant of the shear, S, at this point and H, and its direction is the direction of this resultant. We have, then, from similar triangles, completing the parallelogram on H and S,

Fig. 6.

$$\frac{dx}{dy} = \frac{H}{S} \tag{9}$$

That is, the tangent of the angle which the tangent to the curve at any point makes with the horizontal, is equal to H divided by the shear at that point.

Equilibrium Curve for Uniform Load a Parabola.

Let the distributed load be uniform and equal to p per unit of length. Let the span be $2l$ and the versine at the centre be v. Then since the load is uniform, it is evident that V_1 must equal V_2 and, whatever the form of the curve, each half must be symmetrical. That is, the lowest point of the curve is at the centre, and at this point $\frac{dy}{dx} = 0$.

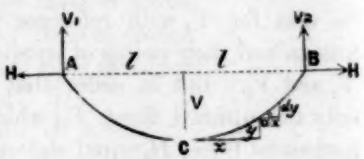

Fig. 7.

Take C as an origin. Then from (9) we have for the vertical force, S, at any point, $S = H\frac{dy}{dx}$. Fig. 7.

The elementary load at any point is then $\frac{dS}{dx} = H\frac{d^2y}{dx^2} = p$.

Integrating this, we have $H\frac{dy}{dx} = px + C$.

Since for $x = 0$, $\frac{dy}{dx} = 0$, the constant $C = 0$ and $H\frac{dy}{dx} = px$.

Integrating again, we have $Hy = \frac{px^2}{2} + C$.

Since for $x = l$, $y = v$, we have $C = Hv - \frac{pl^2}{2}$ and

$$Hy = \frac{px^2}{2} + Hv - \frac{pl^2}{2}.$$

But taking moments about B, we have for the equilibrium of the half BC, since the vertical force at C is zero,

$$Hv = pl \times \frac{l}{2} = \frac{pl^2}{2}$$

Substituting, we have

$$y = v\frac{x^2}{l^2}, \tag{10}$$

which is the equation of a parabola.

Hence, *the curve of a flexible string uniformly loaded is a parabola; and inversely, if a flexible string has the form of a parobola, it must be uniformly loaded.*

Deflection of Cable.—Uniform Live Load Over Entire Span.

Now in the system shown in Fig. 1, the cable carries the entire dead load of the truss, as the suspenders are supposed to be so adjusted during erection, that the unloaded truss just bears at the ends upon the abutments. The weight of truss, flooring, suspenders, etc., may be taken as very nearly uniform, as the variation of weight due to variation of cross-section of flanges and braces can be neglected as very small compared to the uniform dead weight of flooring, wind-bracing, etc. Moreover this uniform load is very great compared to weight of cables. The curve of the cables under the action of the dead load alone may be then considered as very closely a parabola and therefore given by equation (10).

Differentiating, we have $\frac{dy}{dx} = \frac{2vx}{l^2}$.

If the length of arc is s, we have

$$ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

Inserting the value for $\frac{dy}{dx}$

$$ds = dx\sqrt{1+\frac{4v^2x^2}{l^2}} = dx\left(1+\frac{2v^2x^2}{l^4}-\frac{2v^4x^4}{l^8}+\ .....\right)$$

If the ratio $\frac{v}{l}$ is small, as is the case for long spans, we can neglect $\frac{v^4}{l^8}$, and have approximately

$$ds = dx\left(1+\frac{2v^2x^2}{l^4}\right) \qquad (11)$$

Integrating between the limits $-l$ and $+l$, we have for the length of the cable when only the dead load acts,

$$s = 2l\left(1+\frac{2v^2}{3l^2}\right) \qquad (12)$$

Equation (12) gives the original length of cable, when the span and versine are given, and the ratio of $\frac{v}{l}$ is small. That is, for long spans.

Now let the entire span be covered from end to end with the uniformly distributed live load. It is required to find the deflection of the cable.

The new curve will evidently also be a parabola of the same span, whose new length s_1 will be $s_1 = 2l\left(1+\frac{2v_1^2}{3l^2}\right)$ where v_1 is the new versine.

Let λ be the elongation per unit of length at any point caused by the live load acting over the whole span. Now the cable may be composed of links and pins, or of wire. The last is more common. In the first case, the cross-section may be varied according to the strain. The unit strain will then be constant and λ will be constant. In the second case, the cross-section of the cable is constant and equal at any point to the cross-section required by the greatest strain, that is to the cross-section at the ends. The unit strain will then vary as the secant of the angle of inclination, and λ will vary as the unit strain.

Let then λ_1 be the elongation per unit of length at the centre C, Fig. 7. If the cable is composed of links and pins, λ_1 will be constant at every point. If the cable is of wire and of constant cross-section, the elongation per unit of length will be $\lambda_1\frac{ds}{dx}$

From (11) we have then the elongation at any point

$$\lambda = \lambda_1\left(1+\frac{2v^2x^2}{l^4}\right) \qquad (13)$$

We shall find the deflection of the cable for these two cases separately.

1. When λ_1 is constant, or the cross-section of cable varies so that the unit strain is constant.

In this case, the new length of the chain cable will be

$$s+\lambda_1 s = 2l\left(1+\frac{2v_1^2}{3l^2}\right) \text{ or from (12)}$$

$$2l\left(1+\frac{2v^2}{3l^2}\right)(1+\lambda_1) = 2l\left(1+\frac{2v_1^2}{3l^2}\right).$$

From this equation we can find the new versine v_1. Thus

$$v_1 = v\sqrt{1+\lambda_1\left(1+\frac{3l^2}{2v^2}\right)} = v\left[1+\frac{\lambda_1}{2}\left(1+\frac{3l^2}{2v^2}\right)+\frac{\lambda_1^2}{8}\left(1+\frac{3l^2}{2v^2}\right)^2+\ldots.\right]$$

Since λ_1 is a small fraction, this becomes approximately

$$v_1 = v\left[1+\frac{\lambda_1}{2}\left(1+\frac{3l^2}{2v^2}\right)\right] \tag{14}$$

The distance of any point of the cable below the horizontal AB, Fig. 7, is $v-y = v-\frac{vx^2}{l^2}$, where x is the distance of the point right or left of the centre. The distance of the same point of the deflected cable below the horizontal is $v_1-\frac{v_1x^2}{l^2}$.

The difference of these two distances is the deflection at any point, or $\delta_1 = v_1-x-\frac{x^2}{l^2}(v_1-v)$.

But from (14) $v_1-v = \frac{\lambda_1 v}{2}\left(1+\frac{3l^2}{2v^2}\right) = \frac{3}{4}\lambda_1\frac{l^2}{v}\left(1+\frac{2v^2}{3l^2}\right)$ or, since $\frac{v}{l}$ is small for long spans $v_1-v = \frac{3}{4}\lambda_1\frac{l^2}{v}$.

Hence,

$$\delta_1 = \frac{3\lambda_1}{4v}(l^2-x^2) \tag{15}$$

where x is the distance of any point right or left of the centre.

For the deflection at the centre, $x = o$ and

$$\Delta_1 = \frac{3\lambda_1 l^2}{4v} \tag{16}$$

where λ_1 is constant and is the elongation per unit of length due to the strain at the centre caused by the uniform live load when it covers the whole span.

Let the uniform live load be m per unit of length. Then the vertical reactions at the end, Fig. 7, are $V_1 = V_2 = ml$. Taking moments about the centre, we have $Hv_1 = ml \times \frac{l}{2}$ or $H = \frac{ml^2}{2v_1}$ where v_1 is the new versine of the elongated cable. The value of this new versine can be found from (14).

H is the tension at the centre of the cable. If we denote the constant unit strain by f_1 we have from (III)

$$\lambda_1 = \frac{f_1}{E_1} = \frac{H}{F_1 E_1} = \frac{ml^2}{2E_1 F_1 v_1} \tag{17}$$

Equation (15) then becomes

$$\delta_1 = \frac{3ml^2}{8E_1 F_1 v_1 v}(l^2 - x^2) \tag{18}$$

and hence the deflection at the centre of the cable is

$$\Delta_1 = \frac{3ml^4}{8E_1 F_1 v_1 v} \tag{19}$$

where F_1 is the cross-section at the centre of the cable required by the strain at that point, and v_1 is given by the equation

$$v_1 = v\left[1 + \frac{f_1}{2E_1}\left(1 + \frac{3l^2}{2v^2}\right)\right] \tag{20}$$

2. When the Cross-Section of Cable is Constant.—In this case the elongation at any point is from (13) $\lambda = \lambda_1\left(1 + \frac{2v^2x^2}{l^4}\right)$.

The elongation of any element ds of the curve is λds, and the new length of the element is $ds + \lambda ds = ds_1$.

From (11) then, we have

$$ds(1+\lambda) = ds_1 = dx\left(1 + \frac{2v^2x^2}{l^4} + \lambda_1 + \frac{4v^2\lambda_1 x^2}{l^4} + \frac{4v^4\lambda_1 x^4}{l^8}\right).$$

Integrating between $-l$ and $+l$

$$s_1 = 2l(1+\lambda_1)\left(1 + \frac{2v^2}{3l^2} + \frac{2v^2\lambda_1}{3l^2(1+\lambda_1)}\right).$$

Since $\frac{v}{l}$ is a small fraction, and $\frac{\lambda_1}{1+\lambda_1} = \frac{1}{1+\frac{1}{\lambda}}$ is also a very small fraction, we have approximately

$$s_1 = 2l(1+\lambda_1)\left(1 + \frac{2v^2}{3l^2}\right).$$

This is apparently the same length as in the first case. But it must

be remembered that λ_1 or the elongation due to the strain at the centre is now less than before, because the cross-section there is greater, and equal to the cross-section at the ends. Let the cross-section at the ends be F_0. Then, since the cross-section is constant,

$$\lambda_1 = \frac{H}{F_0 E_1} = \frac{ml^2}{2E_1 F_0 v_1}.$$

We have then from (15) for the deflection at any point,

$$\delta_1 = \frac{3ml^2}{8E_1 F_0 v_1 v}(l^2 - x^2) \qquad (18a)$$

$$\Delta_1 = \frac{3ml^4}{8E_1 F_0 v_1 v} \qquad (19a)$$

where we must put for F_0 the cross-section required by the strain at the ends of the cable. Equation (20) gives, as before, the value of v_1.

In *either case*, then, the deflection at any point is

$$\delta_1 = \frac{3ml^2}{8E_1 F v_1 v}(l^2 - x^2) \qquad (IV)$$

If we remember that in the first case F is the cross-section F_0 due to the *strain at the centre*, and in the second case F is the cross-section F_0 due to the strain *at the ends*.

The second case is the most common, as a wire cable is more economical than a chain, and such a cable must have a uniform cross-section. We shall suppose in what follows, therefore, a wire cable of constant cross-section, and therefore $F = F_0$. If the formulæ for the first case are required, we have only to replace F_0 by F_1.

(To be continued.)

Improved Drummond Light.—A Russian naval officer, M. de Robinsky, has improved the lime burners of the Drummond light so as to render them much more refractory, and provided some methods for equalizing the temperature in the different portions, so that a single crayon will last about fifteen days in almost constant service. He proposed to prepare oxygen by the permanganate of potash method, or by a new method which he is now studying, and to supply it to houses, condensed in receivers. Tissandier represents the light as very steady and satisfactory, but he is unable to express any opinion as to its economy.—*La Nature.* C.

STEAMSHIP PERFORMANCE.

By JOHN W. NYSTROM.

The valuable article written by Chief Engineer Isherwood on the performance of the U. S. steamer *Dispatch*, and published in the January number of the JOURNAL, encouraged me to write a similar article to show that the performance of a steamer can be determined beforehand when the drawings of the vessel and machinery are being made.

The various properties of any form and size of a vessel can be treated with correctness by the aid of the parabolic method of constructing ships, and it is here proposed to apply and exemplify that method on the data given by Chief Engineer Isherwood for the *Dispatch*, for the purpose of comparison, and to render the subject more intelligible.

The principal dimensions of the *Dispatch* are as follows:

L = 174 feet, length of the vessel.

B = 25·5 feet, extreme breadth.

d = 12 feet, mean draught of water.

$\mathfrak{M}$ = 186·58 square feet, area of the greatest cross-section of displacement.

$\mathbf{a}$ = 3163·34 square feet, area of the load water line.

D = 19328·64 cubic feet, displacement at 12 feet draught.

0·7138 = ratio of $\mathfrak{M}$ to the surrounding parallelogram.

0·7129 = ratio of $\mathbf{a}$ to the surrounding parallelogram.

0·5945 = ratio of D to $\mathfrak{M}$ L.

0·4250 = ratio of D to the surrounding parallelopipedon.

From these data we find that the exponent for $\mathfrak{M}$ is $n = 2\frac{1}{2}$, which corresponding cross-section is illustrated on Plate VIII, Nystrom's Pocket Book, and the ordinates for that section are obtained from Table I, page 450, where the correct ratio 0·7142 is found in column $\mathbf{a}$ $\mathfrak{M}$ D, opposite the exponent $n = 2\frac{1}{2}$.

The exponent for the displacement is $n = 2\frac{1}{2}$, and power $q = 2$, which make the ratio for $D = 0{\cdot}7138 \times 0{\cdot}5952 = 0{\cdot}4248$, which Chief Engineer Isherwood says is 0·4250.

The number 0·5952 is the ratio of D to $\mathfrak{X}\ L$, and found in column $\mathbf{a}\ \mathfrak{X}\ D$, Table V, page 451, opposite the exponent $n = 2\frac{1}{2}$.

Chief Engineer Isherwood says this ratio is 0·5945.

The centre of gravity of the displacement under the load water line is given to be 7·345 feet, which is evidently wrong, but which can be corrected by formula 4, page 449, N. P. B., namely, as follows:

$$e = \frac{12}{2\left(2 - \frac{19328 \cdot 64}{3163 \cdot 34 \times 12}\right)} = 4 \cdot 0246 \text{ feet, instead of } 7 \cdot 345 \text{ feet.}$$

This formula does not include the displacement of the keels, which lowers the centre of gravity about half an inch in that form and size of displacement.

It now remains to determine the performance of the vessel in motion, that is, the resistance in water, which is equal to the thrust of the propeller, and found by formula 16, page 449, N. P. B. For this formula, it is necessary to find the mean angle of resistance and wet area of the displacement, namely, as follows:

Suppose the section $\mathfrak{X}$ is located at $l' = 109$ feet from the stem, and $l = 65$ feet from the sternpost, then the mean angles of entrance in the bow and delivery in the stern are found by the formulas 17 and 18, page 449, N. P. B., for which the factor t is found in the column *Res. t*, opposite the exponent of the displacement in the tables. In the case before us we have given the exponent of the displacement $n = 2 \cdot 5$, and power $q = 2$, by which we find the tangent $t = 1 \cdot 355$, Table V, page 451.

During the trial, Jan. 14, 1880, the *Dispatch* drew $d = 12 \cdot 2$ feet, making the cross-section $\mathfrak{X} = 195 \cdot 55$, including the sections of the two bilge keels, 4·8 square feet. The cross-section for determining the mean angle of resistance, and wet area of the hull, will then be $\mathfrak{X} = 195 \cdot 55 - 4 \cdot 8 = 190 \cdot 35$ square feet. The area of the load water line during the trial was $\mathbf{a} = 3180$ square feet.

Now we have all the data necessary for calculating the mean angle of resistance, namely, as follows:

Formula 17, page 449. $\text{Tan.} v = \frac{190 \cdot 35 \times 1 \cdot 335}{109 \times 12 \cdot 2} = 0 \cdot 1911.$

Mean angle of resistance, 10° 50′. $\text{Sin.} v = 0 \cdot 18795$. $\sqrt{0 \cdot 18795^3} = 0 \cdot 081483.$

Formula 18, page 449. $\text{Tan.} V = \frac{190 \cdot 35 \times 1 \cdot 335}{65 \times 12 \cdot 2} = 0 \cdot 32045.$

Mean angle of delivery, 17° 46′. Sin. $V = 0{\cdot}30514$. $\sqrt{0{\cdot}30514^3} = 0{\cdot}16856$.

The fractions 0·081483 and 0·16856 are the radicals in the formula 16, page 449, namely, as follows: Resistance

$$R = 2{\cdot}858\ M^2 \left[\mathfrak{X} \left(0{\cdot}9\ \sqrt{\sin.^3 v} + 0{\cdot}1\ \sqrt{\sin.^3 V} \right) + \frac{A\ k}{\sqrt[4]{L}} \right]$$

M = nautical miles per hour, A = wet area of displacement and k = friction coefficient, to be found on page 448, for different conditions of the hull or wet surface of the vessel.

The wet area of the displacement is found by the formula,

$$A = [\mathfrak{a} + 2\ (\mathfrak{X} + d\ L)] \sqrt{\frac{D}{BLd}}$$

The factor under the radical is the ratio 0·4248 of the displacement, and the wet area will be as follows:

$A = [3180 + 2(190{\cdot}35 + 12{\cdot}2 \times 174)]\ \sqrt{0{\cdot}4248} = 5087{\cdot}9$ square feet.

This wet area includes only a part of that of the central keel, but not that of the bilge keels, which Chief Engineer Isherwood says are 416 square feet, to which add 96 square feet for the remainder of the central keel, makes the friction area $A = 5600$ square feet.

The hull of the *Dispatch* was coppered, for which the coefficient of friction is 0·0045. See page 448, N. P. B.

The speed of the vessel on trial was $M = 10{\cdot}75$ nautical miles per hour. Now we have all the data necessary for finding the resistance of the vessel by the above formula 16.

$$R = 2{\cdot}858\ M^2 \left[195{\cdot}55 \left(0{\cdot}9 \times 0{\cdot}081483 + 0{\cdot}1 \times 0{\cdot}16856 \right) + \frac{5600 \times 0{\cdot}0045}{\sqrt[4]{174}} \right] = 8117{\cdot}1 \text{ pounds.}$$

That is, the thrust of the propeller should be 8117·1 pounds. Chief Engineer Isherwood makes the resistance 8146·33 pounds, or only 29·23 pounds more than by my formula. I am convinced that the formula gives the true resistance if supplied with correct data.

The term $\mathfrak{X}(0{\cdot}9\ \sqrt{\sin.^3 v} + 0{\cdot}1\ \sqrt{\sin.^3 V})$ represents the resistance to the vessel in motion, which in the case before us is 17·637, of which $0{\cdot}9\ \sqrt{\sin.^3 v}$ represents the forebody and $0{\cdot}1\ \sqrt{\sin.^3 V}$ the after body

of the displacement. The second term $\frac{A\,k}{\sqrt[3]{L}}$, represents the friction of the wet surface, which in this case is 6·94. The sum of these terms is 24·577, which represents the total resistance. That is, the resistance in displacing the water while the vessel is in motion is 71·762 per cent., and the friction 28·238 per cent. of the gross resistance.

For fuller vessels, the resistance in displacing the water will be much greater in proportion to the friction, but if the vessel is fouled with barnacles the friction may become much greater than the resistance in displacing the water; all of which can be determined by the formula 16 for any form and size of vessel.

Chief Engineer Isherwood says, in the January number of the JOURNAL, pages 25 and 26, that "the entire resistance of the vessel was sensibly that of the water to its immersed external surface; and, consequently, no engine power was expended in overcoming the resistance of the water to displacement by the progress of the vessel. That is to say, the difference between the power exerted by the fore body of the vessel in raising the displaced water from the centre of gravity of the greatest immersed transverse section of the vessel to the general water level, and the power exerted upon the after body of the vessel in the direction of its motion by the ascending column of water caused by the forward movement of the vessel, were sensibly equal," etc.

These ideas conflict with the science of hydraulics.

The horse-power required for friction in the moving of a surface in water is found in the following way:

S = speed or velocity, in feet per second, of the surface through water.

M = nautical miles per hour of the same surface.

k = coefficient of friction, or the force, in pounds, required to move one square foot friction surface with a velocity of one foot per second.

A = friction surface, in square feet.

L = length of the friction surface in the direction of motion.

F = force, in pounds, required to move that surface.

$$F = \frac{AS^2k}{\sqrt[3]{L}}, \text{ of which the horse-power is } \text{HP} = \frac{A\ S^3\ k}{550\ \sqrt[3]{L}}$$

When the velocity is expressed in nautical miles, of 6086 feet each, the force and horse-power will be

Mean angle of delivery, 17° 46′. Sin. $V = 0{\cdot}30514$. $\sqrt{0{\cdot}30514^3} = 0{\cdot}16856$.

The fractions 0·081483 and 0·16856 are the radicals in the formula 16, page 449, namely, as follows: Resistance

$$R = 2{\cdot}858\ M^2 \left[\mathfrak{X} \left(0{\cdot}9\ \sqrt{\sin.^3 v} + 0{\cdot}1\ \sqrt{\sin.^3 V} \right) + \frac{A\ k}{\sqrt[4]{L}} \right]$$

M = nautical miles per hour, A = wet area of displacement and k = friction coefficient, to be found on page 448, for different conditions of the hull or wet surface of the vessel.

The wet area of the displacement is found by the formula,

$$A = [\mathbf{a} + 2\ (\mathfrak{X} + d\ L)] \sqrt{\frac{D}{BLd}}$$

The factor under the radical is the ratio 0·4248 of the displacement, and the wet area will be as follows:

$A = [3180 + 2(190{\cdot}35 + 12{\cdot}2 \times 174)] \sqrt{0{\cdot}4248} = 5087{\cdot}9$ square feet.

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of the displacement. The second term $\frac{A\,k}{\sqrt[3]{L}}$, represents the friction of the wet surface, which in this case is 6·94. The sum of these terms is 24·577, which represents the total resistance. That is, the resistance in displacing the water while the vessel is in motion is 71·762 per cent., and the friction 28·238 per cent. of the gross resistance.

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S = speed or velocity, in feet per second, of the surface through water.

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k = coefficient of friction, or the force, in pounds, required to move one square foot friction surface with a velocity of one foot per second.

A = friction surface, in square feet.

L = length of the friction surface in the direction of motion.

F = force, in pounds, required to move that surface.

$$F = \frac{AS^2k}{\sqrt[3]{L}}, \text{ of which the horse-power is } \text{HP} = \frac{A\ S^3\ k}{550\ \sqrt[3]{L}}$$

When the velocity is expressed in nautical miles, of 6086 feet each, the force and horse-power will be

$$F = \frac{6086^2\ A\ M^2\ k}{(60\times 60)^2\ \sqrt[4]{L}} = \frac{10288{\cdot}7\ A\ M^2\ k}{\sqrt[4]{L}}$$

$$\text{HP} = \left(\frac{6086}{60\times 60}\right)^3 \frac{A\ M^3\ k}{550\ \sqrt[4]{L}} = \frac{A\ M^3\ k}{113{\cdot}935\ \sqrt[4]{L}}$$

Example.—Required the horse-power of a smooth copper surface, $A = 5698$ square feet, for which $k = 0{\cdot}0045$, moving with a speed of $M = 10{\cdot}75$ nautical miles per hour, when the length $L = 174$ feet.

$$\text{HP} = \frac{5600\times 10{\cdot}75^3\times 0{\cdot}0045}{113{\cdot}935\times\sqrt[4]{174}} = 75{\cdot}653 \text{ horse-power.}$$

Chief Engineer Isherwood says 266 horse-power.

The horse-power required for propelling the steamer *Dispatch* at a speed of $M = 10{\cdot}75$ nautical miles per hour, with the resistance given by the formula 16, should be

$$\text{HP} = \frac{6086\ M\ R}{60\times 33000} = \frac{10{\cdot}75\times 8117{\cdot}1}{325{\cdot}337} = 268{\cdot}21 \text{ horse-power.}$$

Chief Engineer Isherwood says 269·17 horse-power.

The horse-power consumed by friction of the water against the propeller blades is found by the following formula:

$$\text{HP} = \frac{k\ \mathbf{R}\ L\ N\ n^3}{59400000\ P}\ (311{\cdot}71\ \mathbf{R}^4 + 26{\cdot}319\ \mathbf{R}^2\ P^2 + P^4)$$

See Nystrom's "Elements of Mechanics," pages 178 to 181 inclusive, for explanation how this formula is obtained.

For the propeller of the steamer *Dispatch* we have given $\mathbf{R} = 5{\cdot}55$ feet radius; $L = 1{\cdot}4583$, the length in the direction of its axis; $P = 19{\cdot}9$ feet pitch; $N = 4$ blades; $n = 64{\cdot}533$ revolutions per minute; $k =$ friction coefficient, which for smooth finished brass is 0·0045. The friction horse-power will then be

$$\text{HP} = \frac{0{\cdot}0045\times 5{\cdot}55\times 1{\cdot}4583\times 4\times 64{\cdot}5333^3}{59{,}400{,}000\times 19{\cdot}9}\ (311{\cdot}71\times 5{\cdot}55^4 + 26{\cdot}319 \times 5{\cdot}55^2\times 19{\cdot}9^2 + 19{\cdot}9^4) = 26{\cdot}73 \text{ horse-power.}$$

Chief Engineer Isherwood gives the friction horse-power to be 25·62, with the friction $k = 0{\cdot}0045$, for which the same formula has probably been used. This savant, I believe, was the first one to appreciate the value of and to determine the friction horse-power of screw propellers, and his first process was very laborious. See JOURNAL OF THE FRANKLIN INSTITUTE, February, 1854, page 121.

To find the horse-power required for driving a vessel a given speed.

For this purpose, like for the resistance, it is necessary to know the shape of the vessel, that is, its exponents, angles of resistance, greatest immersed section, wet area of the hull, etc.

$$\text{Call } [X] = \left[\mathfrak{A} \left(0{\cdot}9 \sqrt[3]{\sin.^3 v} + 0{\cdot}1 \sqrt[3]{\sin.^3 V} \right) + \frac{A\,k}{\sqrt[3]{L}} \right]$$

This is the factor of resistance, which in the case of the *Dispatch* is $[X] = 24{\cdot}577$.

The horse-power required for driving a vessel of any form and size will be as follows:

$$\text{HP} = \frac{M\ R}{325{\cdot}337} = \frac{2{\cdot}858\ M^3}{325{\cdot}337}\,[X]$$

By eliminating the coefficient 2·858, the formula for horse-power becomes

$$\text{HP} = \frac{M^3}{113{\cdot}82}\,[X]$$

The indicated horse-power of the engine should be about 50 per cent. more for working the pumps and frictions of the engine and propeller.

The speed with which a steamer will run with a given horse-power will be

$$M = \sqrt[3]{\frac{113{\cdot}82\ \text{HP}}{[X]}}$$

The steamer *Dispatch* is constructed very near like the steam propeller represented on Plate X, N. P. B., of which the general formula appears on page 446. A general formula for the *Dispatch* may be set up as follows:

$$\left\{ \frac{W\ 2{\cdot}5\times 1}{D\ 2{\cdot}5\times 2} \right\} 65 \left(\frac{174\times 25\times 12}{\mathfrak{A}\ 2{\cdot}5\times 1} \right) 109 \left\{ \frac{W\ 2\times 1{\cdot}5}{D\ 2{\cdot}5\times 2} \right\}$$

Any one who understands the parabolic method can construct a vessel like the *Dispatch* from the above formula, and he can know all the properties of the vessel before a single line of it is laid down for the drawing. The parabolic method enables the shipbuilder to reason with certainty and clearness about forms of ships.

The order of proceedings in the ordinary or old method of constructing ships is as follows:

Some boards are planed up, and put together with wooden pins,

from which a model of the intended shape of the vessel is carved out according to the rule of thumb judgment, and several models are often made before a desired shape is obtained. When the model is finished it is taken apart, and each board is laid on the drawing paper for drawing the corresponding water lines therefrom. After all the water lines are so drawn, the body plan is made therefrom.

From this drawing the lines were laid down on the ship's floor by the aid of scale of measurements. If it is desired to know correctly the properties of the vessel, such as areas of water lines and cross-sections, displacement, centres of gravity, metacentre, etc., calculations are made by the rules of Chapman or Simpson, which is a very laborious process, very seldom carried out, but the shipbuilders rely upon their experience in approximating these data sufficiently near for practical purposes.

With the parabolic method of constructing ships the order of proceedings is the reverse of that of the old method, namely, as follows:

First, set up a formula for the desired shape of the vessel, from which calculate the ordinates, areas of water lines and cross-sections, displacement, centres of gravity metacentre, etc. From the so obtained data lay down the lines on the drawing for the complete vessel. If a model is required make it from the drawing. Then lay down the lines on the ship's floor, not altogether by scale measurements, but from the original calculation, because the scale measurements are not reliable when enlarged from the drawing to the full size of the ship.

The calculations required by the parabolic method is about one-tenth of that by the Chapman's or Simpson's rules.

Transit Observations.—M. Virlet D'Aoust has proposed to M. Dumas, the President of the International Commission upon the Transit of Venus, a plan for preventing the disturbances of irradiation. It consists of an eclipsing disc, or diaphragm, which is connected with clockwork so as to move through the field of the telescope with the same rapidity as the planet. The luminous phenomena being thus withdrawn from the eyes of the observers, he thinks that they could better appreciate the precise moments of contact, so that Halley's method could be practically applied and an approximation of the solar parallax obtained which would be much more satisfactory than was possible at any previous transit.—*Les Mondes.* C.

RADIO-DYNAMICS: ATOMIC PHYLLOTAXY AND KINDRED HARMONIES.

By Pliny Earle Chase, LL.D.

Communicated to the American Philosophical Society, November 4, 1881.

Clarke (*P. Mag.* [5] xii, 109–110) gives the result of his recalculation of atomic weights, which inclines him to look favorably on Prout's hypothesis, although he had previously believed that it had been forever overthrown. Maximilian Gerber (*Les Mondes*, cited in *Chemicel News*, xliii, 242–243) rejects the hypothesis, but he gives four additional empirical units, which seem to show that groups of similar valency may have special common divisors. The varied evidences of the phyto-dynamic importance of hydrogen favor Clarke's deliberate opinion, and Gerber's factors may help towards its establishment. The possibility of measuring undulating *vis viva* by orbital areas, as well as by the distance of projection against uniform resistance, gives a clue for reconciling some apparent oppositions of indications.

The phyllotactic law distributes leaves and branches evenly around the stems of vegetables, so that all parts of the plant may share the benefit of heat, air and moisture. In 1849, Dr. Thomas Hill, at the request of Professor Peirce, showed that the times of planetary revolution are phyllotactic (*Proc. Amer. Assoc.*, vol. 2). The planets are, therefore, distributed around the sun so evenly as to avoid the destruction of the system by the accumulated perturbations of its principal masses.

If the several atomic elements have especial systems of ætherial vibrations, we may reasonably look for evidences of a phyllotactic harmony which contributes to the stability of equilibrium in chemical compounds. The following table, which includes about one-half of the known elements, contains multiples of the phyllotactic divisor, $\frac{8}{5}$H or 1·6H, compared with Clarke's recalculation of atomic weights.

	Phyllotactic.		Clarke.	Difference.
O	$10 \times 1{\cdot}6 =$	16	15·963	·037
Fl	$12 \times 1{\cdot}6 =$	19·2	18·984	·216
Mg	$15 \times 1{\cdot}6 =$	24	23·951	·049

	Phyllotactic.	Clarke.	Difference.
S	20 × 1·6 = 32	31·984	·016
Cl	22 × 1·6 = 35·2	35·370	·170
Ca	25 × 1·6 = 40	39·990	·010
Ti	31 × 1·6 = 49·6	49·847	·246
V	32 × 1·6 = 51·2	51·256	·056
Se	49 × 1·6 = 78·4	78·797	·397
Br	50 × 1·6 = 80	79·768	·032
Zr	56 × 1·6 = 89·6	89·367	·234
I	79 × 1·6 = 126·4	126·557	·157
Te	80 × 1·6 = 128	127·960	·040
Cs	83 × 1·6 = 132·8	132·583	·217
Ytter	108 × 1·6 = 172·8	172·761	·039
Bo	7 × 1·6 = 11·2	10·940	·260
Al	17 × 1·6 = 27·2	27·009	·191
Fe	35 × 1·6 = 56	55·913	·087
Ga	43 × 1·6 = 68·8	68·854	·054
Cd	70 × 1·6 = 112	111·770	·230
In	71 × 1·6 = 113·6	113·398	·202
Yt	56 × 1·6 = 89·6	89·816	·216
Ru	65 × 1·6 = 104	104·217	·217
Ro	65 × 1·6 = 104	104·055	·055
Rh	66 × 1·6 = 105·6	105·737	·137
Sb	75 × 1·6 = 120	119·955	·045
Ta	114 × 1·6 = 182·4	182·144	·256
W	115 × 1·6 = 184	183·610	·390
Os	124 × 1·6 = 198·4	198·494	·094
Hg	125 × 1·6 = 200	199·712	·288
Pb	129 × 1·6 = 206·4	206·471	·071
Th	146 × 1·6 = 233·6	233·414	·186
U	149 × 1·6 = 238·4	238·482	·082

The greatest difference in the above table of 33 elements is less than 25 per cent. of the phyllotactic unit. Gerber's longest table for a single divisor contains but 25 elements; his greatest difference is more than 36 per cent. of his empirical divisor. If the eight elements which possess the greatest phyllotactic differences were rejected from this comparison, so as to make the table of the same length as Gerber's, the greatest remaining difference would be less than 15 per cent.

of the divisor. The photo-dynamic approximation is, therefore, much closer than the empirical.

Gerber says that "no single relation exists among" his divisors, therefore they "have no value in themselves." There is, however, a relation he failed to discover, for they also are phyllotactic, as will be seen by the following comparison:

Gerber.		Phyllotactic.	
H	·9997	H	·998
D_1	·768	$\frac{5}{13} \times 2H$	·768
D_2	1·995	2H	1·996
D_3	1·559	$\frac{5}{2} \times \frac{5}{8}H$	1·559
D_4	1·245	$\frac{5}{2} \times \frac{1}{4}H$	1·247

The first six phyllotactic numbers are 1, 2, 3, 5, 8, 13; the third does not appear in the formation of the theoretical divisors, but the others are all employed. The simple phyllotactic relation of all the divisors to H shows that they *have* "value in themselves." Upon examining their mutual relations, it will be seen that $D_1 = \frac{5}{13}D_2 = 2 \times \frac{2}{5} \times \frac{8}{13}D_3 = \frac{8}{13}D_4$; $D_2 = 2 \times \frac{2}{5} \times \frac{5}{5}D_3 = \frac{5}{5}D_4$; $D_3 = \frac{5}{2} \times \frac{1}{2}D_4$; $D_4 = 1\sqrt{HD_3}$. These varied provisions for the stability of cyclical equilibrium, in all possible varieties of intermolecular ætherial movements, show that the command, "Let there be light," manifested its formative power of organization as soon as material atoms were set in motion.

The appearance of the first five phyllotactic numbers in crystallization, furnishes a step from inorganic to organic morphology, giving new meaning to the landscapes on our frosted window-panes, as well as to the protective mimicry of vegetables and animals, as illustrations of the "distributive ratio" which alike controls light-waves, atomic inertia, crystalline structure, organic growth, planetary configuration and interstellar action.

The importance of oxygen and hydrogen, both in mutual combination and in connection with other elements, suggests the following comparative grouping of Clarke's table of atomic weights:

O = 16; H = 1·0932.		Difference.	O = 16; H = 1·0023.		Difference.
Br	79·951	·049	Bi	208·001	·001
I	126·848	·152	Pb	206·946	·054
Mg	24·014	·014	Mn	54·029	·029
Zn	65·054	·054	Fe	56·042	·042
Cs	132·918	·082	Ni	58·062	·062

O = 16; H = 1·0023.		Difference.	O = 16; H = 1·0023.		Difference.
Ag	107·923	·077	Co	59·023	·023
Tl	204·183	·183	Bo	10·966	·034
Se	78·078	·022	Ga	68·963	·037
Mo	95·747	·253	Ce	149·747	·253
W	184·032	·032	Ytte	90·023	·023
U	239·030	·030	Ytter	173·158	·158
P	31·029	·029	La	138·844	·156
Cd	112·027	·027	Di	144·906	·094
Hg	200·171	·171	Th	233·951	·049
Ba	137·007	·007	Pt	194·867	·133
C	12·001	·001	Ir	193·094	·094
Ti	49·961	·039	Os	198·951	·049
Sn	117·968	·032	Pd	105·981	·019
Fl	18·984	·016	Zr	89·367	·367
Cl	35·370	·370	N	14·021	·021
Li	7·007	·007	Sb	115·955	·045
Gl	9·085	·085	Ta	182·144	·144
Na	22·998	·002	Sc	43·980	·020
K	39·019	·019	Al	27·009	·009
Rb	85·251	·251	V	51·256	·256
S	31·984	·016	As	74·918	·082
Te	127·960	·040	Cu	63·173	·173
Cr	52·009	·009	Er	165·891	·109
Ca	39·990	·010	Rh	104·055	·055
Sr	87·374	·374	Ru	194·217	·217
Si	28·195	·195	Au	196·155	·155

The above tables seem to show that, if Prout's law is correct, the value of the oxygen atom has been more accurately determined than that of the hydrogen atom. The deviations of Mo, In, Ce, Cl, Rb, Sr, Zr and V are so great as to require some explanation, which may, perhaps, be found in phyllotactic or harmonic influence, as indicated by the following relations to oxygen:

	Observed.		Phyllotactic.	Difference.
Mo	95·747	$\times \frac{1}{16}$O	$2^5 \times 3 = 96$	·253
In	113·659	$\times$ "	$7 \times 13 \times \frac{5}{4} = 113{\cdot}75$	·091
Ce	140·747	$\times$ "	$3 \times 61 \times \frac{10}{13} = 140{\cdot}769$	·022
Cl	35·451	$\times$ "	$2 \times 23 \times \frac{10}{13} = 35{\cdot}385$	·066
Rb	85·529	$\times$ "	$3 \times 37 \times \frac{10}{13} = 85{\cdot}385$	·144

	Observed.		Phyllotactic.		Difference.
Sr	87·575 ×	"	$5^2 \times 7 \times \frac{1}{2} =$	87·5	·075
Zr	89·573 ×	"	$11 \times 13 \times \frac{5}{8} =$	89·375	·198
V	51·373 ×	"	$2 \times 41 \times \frac{5}{8} =$	51·25	·123

"Sir W. THOMSON is led, from the consideration of various experiments with fluids and solids and the study of smoke rings, to speculate upon elasticity as an evidence of motion. The kinetic theory of gases requires that the molecule or atom shall be elastic. 'But this kinetic theory of matter is a dream and must remain so until it can explain chemical affinity, electricity, magnetism, gravitation and inertia.' The writer looks forward to a greater generalization which shall include elasticity as a form of motion."—J. T., in *Amer. Jour. of Science*, Nov. 1881.

My first physical paper (*Proc. Am. Phil. Soc.*, ix, 283–8) deduced approximate values of solar mass and distance from the combined action of daily rotation, yearly revolution and atmospheric elasticity. All my subsequent radio-dynamic investigations have been based upon the consideration of the various forms of harmonic relation which *ought* to follow from the undulations of an all-pervading elastic medium, such as the luminiferous æther is generally supposed to be.

Schuster (*Proc. Roy. Soc.*, xxxi, 337–47) discusses the probability of accidental harmonic coincidences in spectral wave-lengths, giving the following summary of his results for the iron spectrum:

"*1. There is a real cause acting in a direction opposed to the law of harmonic ratios, so far as fractions formed by numbers smaller than seventy are concerned.*

"*2. After elimination of the first cause, a tendency appears for fractions formed by two lines to cluster round harmonic ratios.*

"*3. Most probably some law hitherto undiscovered exists, which in special cases resolves itself into the law of harmonic ratios.*"

The relations which I have pointed out, between planetary harmonic roots and spectral harmonic quotients, suggest the probability that the opposition to strict harmonic ratios may be due to differences of inertia in the wave-systems, which would be more rigidly harmonic were it not for such differences. The simple tendency of all elastic media to harmonic vibrations would then be the general law, instead of a law which becomes harmonic "in special cases."

In waves which are propagated with such rapidity as those of light it seems reasonable to suppose that there may be large harmonic fac-

tors, which are modified by smaller disturbing influences. Schuster's analysis does not reach my own harmonic comparisons. In the investigation which was suggested by Dr. Henry Draper I found 11 harmonic divisors with a common cosmical factor, which deviate from the observed divisors by a mean amount of less than $\frac{1}{19}$ of one per cent. The greatest difference is in the C line, where the theoretical harmonic divisor is 1·1530, the observed divisor being 1·1592, giving a deviation of $\frac{31}{58}$ of one per cent. The greatest difference between either of my harmonic lines and the corresponding "basic line" is $\frac{1}{3}$ of one per cent. In my harmonic relations of the corona line the greatest discrepancy is $\frac{1}{20}$ of one per cent; in those of the hydrogen lines $\frac{1}{14}$ of one per cent.

Liveing and Dewar (*Proc. Roy. Soc.*, xxxii, 189–203) give some results of their investigations on the spectrum of magnesium, which seem to strengthen the probability of large harmonic factors, modified by small disturbances. The only two single lines which are found in the flame-, arc- and spark-spectra have wave-lengths of 2850 and 4570, respectively. These represent, with close approximation, the phyllotactic numbers 5 and 8, viz.:

Phyllotactic.	Observed.
2854	2850
4566	4570

The phyllotactic ratio $\frac{5}{8}$ also appears between the limiting lines of the hydrogen spectrum C and *h*.

In the arc- and spark spectra there is "a very striking group of two very strong lines at wave-lengths about 2801 and 2794" (*loc. cit.*, p. 201), and "one line common to the arc and spark at wave-length 4703" (*Ib.*, p. 203), which does not appear in Angstrom's table." The difference, 7, between the "two very strong lines" has modified the other lines, as is shown below:

Harmonic.	Observed.
$7 \times 399 = 2793$	2794
$7 \times 400 = 2800$	2804
$7 \times 407 = 2849$	2850
$7 \times 653 = 4571$	4570
$7 \times 672 = 4704$	4703

The greatest discrepancy is $\frac{1}{7}$ of the harmonic unit, the mean difference being only $\frac{1}{35}$ of the unit. If there were no law controlling the

approximations, the probable maximum difference would be ·5 of the unit, and the mean difference would be ·25 of the unit.

It seems reasonable to look for stronger evidence of undisturbed or slightly modified harmonic influence in hydrogen than in any of the heavier elements. Accordingly, Professor Johnstone Stoney has "shown that three out of the four lines in the visible part of the spectrum have wave-lengths which, to a high degree of accuracy, are in the ratio of 20:27:32." I find, moreover, that three of the lines are in simple geometric ratio, as will be seen by the following comparisons:

	Theoretic Harmonic Lines.	Observed.
$\alpha = (2 \times 3)^3 \times 30{\cdot}379 =$	6562·8	6561·8
$\beta = 5 \times 2^5 \times 30{\cdot}379 =$	4860·6	4860·6
$\gamma = (\delta^2 \beta)^{\frac{1}{3}} =$	4340·1	4340·
$\delta = 5 \times 3^3 \times 30{\cdot}379 =$	4101·1	4101·2

The extreme lines are phyllotactic, δ being $\frac{5}{8}$ of α. The greatest discrepancy is $\frac{1}{4\,1\,0}$ of one per cent., which is so far within the limits of probable error as to furnish ground for the following examination of Schuster's criteria.

1. The ratio between the hydrogen lines α and δ is between $\frac{6}{5}$ and $\frac{3}{2}$.

$\alpha \div \delta = a =$	1·59997
$8 \div 5 = b =$	1·60000
$3 \div 2 = c =$	1·50000
$b - c = d =$	·10000
$b - a = e =$	·00003

$e \div d = \frac{3}{1\,0\,0\,0\,0\,0}$, or less than $\frac{1}{8\,3}$ of one per cent. of the probable error.

2. The ratio between γ and δ is between $\frac{2\,1}{8\,6}$ and $\frac{1\,8}{1\,7}$.

$\gamma \div \delta = a_1 =$	1·058227
$18 \div 17 = b_1 =$	1·058824
$91 \div 86 = c_1 =$	1·058140
$b_1 - c_1 = d_1 =$	·000684
$a_1 - c_1 = e_1 =$	·000087

$e_1 \div d_1 = \frac{2\,9}{2\,2\,8}$, the probable error being $\pm \frac{5\,7}{2\,2\,8}$.

3. The ratio between α and β is between $\frac{2\,7}{2\,0}$ and $\frac{7\,7}{5\,7}$.

$\alpha \div \beta = a_2 =$	1·350165
$77 \div 57 = b_2 =$	1·350877
$27 \div 20 = c_2 =$	1·350000
$b_2 - c_2 = d_2 =$	·000879
$a_2 - c_2 = e_2 =$	·000165

$e_2 \div d_2 = \frac{1\,6\,5}{8\,7\,7}$, the probable error being $\pm \frac{2\,1\,9}{8\,7\,7}$.

If we were to end our comparisons at this point we might pronounce the test entirely satisfactory, and the evidence of harmonic influences, in which all the lines are involved, would be deemed conclusive. But if we try another mode of grouping we reach a different result.

4. The ratio between β and δ is between $\frac{32}{27}$ and $\frac{77}{65}$.

$\beta \div \delta = a_3 =$	1·185018
$77 \div 65 = b_3 =$	1·184616
$32 \div 27 = c_3 =$	1·188185
$c_3 - b_3 = d_3 =$	·000569
$c_3 - a_3 = e_3 =$	·000166

$e_3 \div d_3 = \frac{166}{569}$, the probable error being only $\frac{142}{569}$. Therefore the test fails to discover any harmonic influence in this case.

5. The ratio between β and γ is between $\frac{28}{25}$ and $\frac{75}{67}$.

$\beta \div \gamma = a_4 =$	1·119816
$28 \div 25 = b_4 =$	1·120000
$75 \div 67 = c_4 =$	1·119403
$b_4 - c_4 = d_4 =$	·000507
$b_4 - a_4 = e_4 =$	·000184

$e_4 \div d_4 = \frac{184}{597}$, the probable error being only $\pm \frac{142}{597}$. Therefore the test fails in this case also.

6. The ratio between α and γ is between $\frac{62}{41}$ and $\frac{65}{43}$.

$\alpha \div \gamma = a_5 =$	1·511936
$62 \div 41 = b_5 =$	1·512195
$65 \div 43 = c_5 =$	1·511698
$b_5 - c_5 = d_5 =$	·000567
$b_5 - a_5 = e_5 =$	·000259

$e_5 \div d_5 = \frac{37}{81}$, the probable error being only $\pm \frac{20}{81}$. The test, therefore, fails again, the number of failures in the whole comparison being exactly equal to the number of confirmations. Hence it is evident that Schuster's criterion is insufficient, at least when the probable errors of observation are not satisfactorily ascertained. Even when they are known, the proper application of the test will often require supplementary calculations of such intricacy as to make it practically inoperative.

By increasing the magnitude of the harmonic ratios the test may sometimes be made to indicate a probability, which really exists, but which is not shown by Schuster's method. For example, $\beta \div \gamma$ is between $\frac{6}{5}$ and $\frac{7}{6}$. These values give $d_4 = ·033333$; $e_4 = ·000184$; $e_4 \div d_4 = \frac{134}{33333}$, which is less than $\frac{1}{45}$ of the probable error. In like manner, $\alpha \div \gamma$ is between $\frac{3}{2}$ and $\frac{4}{3}$. These values give $d_5 = ·166667$; $e_5 = ·011936$; $e_5 \div d_5 = \frac{1119}{15625}$, which is less than $\frac{4}{13}$ of

the probable error. These results seem to indicate the propriety of harmonic comparisons between terms which are unquestionably of the same order of magnitude. Thus, in Schuster's calculation (*loc. cit.*, p. 338), the ratio ·96476 lies between $\frac{55}{57}$ and $\frac{55}{58}$, the difference between these two fractions being ·016636. The difference of the fraction in the sodium spectrum from the nearest of the comparative fractions is ·000152, which is only ·00914 of the difference between the fractions themselves, or less than $\frac{1}{27}$ of the probable error.

If a supposed harmonic relation can be represented by a fraction with terms of a single digit, Schuster's test might fail even with the above modification, provided the probable error of observation should be greater than $\frac{1}{4} \times \frac{1}{8} \times \frac{1}{9}$; if the terms are of two digits it would not be trustworthy if the probable error were greater than $\frac{1}{4} \times \frac{1}{98} \times \frac{1}{99}$. If the modifications of *vis viva* in synchronous wave-systems are of the same order of magnitude as the variations of planetary eccentricity, the limit of probable error would be at least $\frac{1}{3}$, instead of $\frac{1}{4}$ of the difference between adjacent fractions which have a common numerator. This would be the case for each of the compared pairs of wave-lengths, the probability for the entire system being equivalent to the product of the independent probabilities.

All estimates of abstract probability, in such cases, should be greatly increased by the mathematical necessity that synchronous undulations in elastic media *must be* harmonic. In view of this consideration, the indications of a harmonic tendency pervading an entire system, such as I have pointed out in many of my own comparisons, are far more significant than conclusions which can be drawn from restricted investigations.

Whatever tests may be applied, it should always be remembered that the failure to discover a harmonic influence between any two given lines does not affect, in the slightest degree, the evidence of harmonic influence between other lines. The failing cases are entitled to no weight in drawing the final conclusion. We should, therefore, have been fully justified in stopping our examination of the observed lines in the hydrogen spectrum as soon as we found that α is harmonically connected with β and δ, and that γ is similarly connected with δ. Even if subsequent discussion had failed to show any probable evidence of harmony between β and γ, β and δ, α and γ, the fact that there are such harmonies, operating through the relations of the intermediate to the extreme wave-lengths, would have been unshaken.

A New Seaport.—It has been proposed in Germany to dredge the Rhine, so that Cologne may become a seaport. The proposition has been received with much favor and with some apprehension upon the part of the Hollanders and Netherlanders, through fear of serious injury to the ports of Antwerp and Rotterdam.—*Chron. Indust.* C.

Tunnel through the Pyrenees.—Louis XIV, when sending his grandson to reign over the Spaniards, said: "There are no more Pyrenees." The Spaniards propose to give a new meaning to his boast by tunneling the mountains, in order to remove the obstacle which they oppose to commercial relations with France. A bill has already been presented to the Cortes for authorizing the undertaking. The expenses are to be borne by the two countries. The tunnel would shorten the distance between Madrid and Paris about 100 kilometres (62·14 miles).—*Chron. Indust.* C.

Columnar Lightning.—Prof. F. Mandoj, of Albano, describes a meteorological phenomenon which he observed at Noci, a village between the Adriatic and the Gulf of Tarentum. The sky, which had been clear the whole morning, with a cool and pleasant breeze, began to darken soon after noon, with clouds indicating a storm from the south. About 4 o'clock other clouds came rapidly from the northwest, from which a gentle rain fell. The south wind soon increased, driving its clouds before it and accompanied by a violent rain and numerous terrible peals of thunder. When the clouds from the south passed those from the northwest the flashes passed between the strata of cloud so rapidly and yet so silently as to produce the appearance of majestic columns of light, lasting in some cases as much as 7 seconds. Prof. Mandoj supposes that the two clouds were charged with electricity of the same kind, but as they were moving in different directions they might be compared to an immense electrical machine, of which the upper cloud was the conductor, the lower the disc, and the wind the motive force. The moisture of the air served as a conductor for a continuous silent discharge of the electricity that was induced by the motion of the clouds. In order to test his theory, he connected an electrical machine with an insulated disc. Upon electrifying the disc with electricity of the same kind as that of the machine and moistening the air between the disc and the ground, while the machine turned rapidly in a warm and dry air, he produced columnar flashes similar to those of the storm.—*Les Mondes.* C.

Franklin Institute.

HALL OF THE INSTITUTE, Jan. 18, 1882.

The stated meeting was called to order at 8 o'clock P.M.

There were present 124 members and 20 visitors.

The minutes of the last meeting were read and approved.

The Actuary presented the minutes of the Board of Managers and announced that at the last meeting of the Board 25 persons were elected members of the Institute.

The Actuary read the following Report of the Board, which on motion was adopted:

ANNUAL REPORT OF THE BOARD OF MANAGERS.

The Board of Managers of the Franklin Institute of the State of Pennsylvania, for the Promotion of the Mechanic Arts, respectfully reports for the past year of 1881.

Members.—During the year 200 members have been elected, and 26 have resigned.

Treasurer's Report.—The following is a condensed summary of the Treasurer's report for the year ending December 31, 1881:

Receipts.

Balance on hand Jan. 1, 1881,	$1,510 01	
Investments paid to Institute,	1,000 00	
Receipts from all other sources,	12,839 96	
		$15,349 97

Payments.

Amount re-invested,	$820 00	
All other current payments,	13,240 33	
Balance, Dec. 31, 1881,	1,325 64	
		$15,349 97
Showing decrease in money invested,		$180 00
decrease in cash,		184 37
		$364 37

The increased value of the property of the Institute, held for the promotion of its legitimate objects, far exceeds this sum.

Library.—The Board estimates the annual increase of the money value of the Library for the last five years at two thousand dollars, or ten thousand dollars in all, and the design is to continue the improvement at the same rate.

The Board of Managers call especial attention to the complete records of scientific discovery and inventions contained in the Library. In this respect we possess a collection which is probably unequalled in value by any other similar collection in this country. The practical recognition of this fact is shown by the frequent use which is made of the Library by persons visiting it from all parts of the country.

Your Committee on the Library will present the annual report of matters under its charge. The want of room for the proper arrangement of books is increasingly felt.

Journal.—There is no change to note in the management of the JOURNAL, except that we now print a larger edition of the increased size noticed in the last annual report.

Lectures.—In the beginning of the year Prof. Edward D. Cope delivered a course of four illustrated lectures on Geology, followed by two lectures from Prof. Persifor Frazer on "Modern Methods in Geological Research"; two from Mr. Hector Orr on the "Rise and Progress of Manufactures in Philadelphia;" four from Prof. Henry Hartshorne on Hygiene; a lecture from Mr. Alexander E. Outerbridge, Jr., on the "Fourth State of Matter," illustrated with Prof. Crookes' apparatus; two from Mr. D. S. Holman on the "Microscope and its Revelations"; two from Prof. Pliny E. Chase on Astronomy; two from Mr. Robert Briggs on "Machine Design and Construction"; two from Mr. W. B. Cooper on "Drainage and Disease and Utilization of Sewage"; a lecture from Mr. John Carbutt on "Modern Photography and Gelatine Prints"; a lecture from Dr. Samuel Chamberlaine on "Silk and its Culture", and one from Mr. Robert Grimshaw on Electrotyping.

In November the course was opened by four lectures from Mr. C. Henry Roney on Civil and Mining Engineering; a lecture by Mr. Coleman Sellers, Jr., on Mechanical Drawing; one from Theodore D. Rand, Esq., on the Microscope and Polarized Light in Mineralogy, and one from Mr. John H. Ryder, of the United States Fish Commission, on the Growth and Structure of the Oyster. A course of six illustrated lectures upon Mechanics by Prof. S. T. Skidmore, and eight experimental ones on Modern Chemistry by Mr. Henry Trimble were also commenced early in December, and are now being delivered to highly appreciative audiences. Mr. D. S. Holman also gave, during the Christmas holidays, a lecture to young people on the "Wonders of the Microscope," the Hall not being able to contain all that wished to attend.

The Board of Managers, at their meeting in October, appointed, upon the recommendation of the Committee on Instruction, Coleman Sellers to be Professor of Mechanics; Edwin J. Houston to be Professor of Physics, and Persifor Frazer to be Professor of Chemistry. As an advisory board they have rendered already very valuable services to the Committee on Instruction in the selection and arrangement of the courses in their respective departments this winter. The joint committee decided, with the approval of the Board, to give to members the additional privilege of bringing a friend with them at any time to the lectures. Complimentary tickets, admitting at 5 minutes to 8 o'clock, have also been prepared to be distributed by members and the officers at the Institute, so that, practically, the lectures are now, as last year, free to the public. The average attendance, however, has not been so great this year, but the members inviting the audience is thought to be an improvement upon the former plan, and those who have attended the lectures have expressed themselves as much pleased with it. Important courses are still to be delivered on Astronomy, Chemistry Applied to the Arts, and Physics. Members will do well to attend these lectures, among the many privileges they have in connection with the Institute.

Drawing School.—No work connected with the Institute has shown greater progress than this, while the number of pupils enjoying the advantages offered by the thorough instruction in this department has been nearly double that of last year, taxing the resources of the Institute and its school rooms to the utmost.

Early in the autumn the resignation of Mr. Philip Pistor from the direction of the school, owing to other engagements, necessitated the choice of a successor, and the committee were fortunate in securing Mr. William H. Thorne to fill the position which had been so ably done by Mr. Pistor. The same general plan of instruction has been followed, with some few changes that experience had shown would be desirable, and the course of instruction extending over two years is a progressive one, as formerly, the classes, however, being frequently re-arranged according to the relative ability and success the pupils display.

Particular attention has been given to the technicalities of drawing, such as the proper selection, the use and care of instruments and materials, the making of clear and perfect lines in pencil and ink, the use of the brush and colors, the relative arrangement of the different

views of an object, and other conventionalities of practical draughting. Instruction has also been given in the modern practice of projections, the plotting of mechanical movements and gearing, free-hand sketching from models and patterns, and, in short, as thorough instruction in mechanical drawing, based upon practical methods, as the time will allow. Special classes in architectural and machine drawing have also been started, but the regular course is the one recommended to the student.

Certificates of merit will also be given at the end of the course and the Bartol scholarships of free instruction in the second term awarded to twelve of the most deserving pupils during the first term.

This drawing school has now been carried on for more than half a century, and the Board of Managers are not only unanimously agreed upon its great importance, but that it represents to an eminent degree the useful work of the Institute.

Sections.—During the last year Mechanical and Chemical Sections were established, composed of members interested in these branches, at which original papers are presented and discussed. The Phonetic Section is also kept up, and the Actuary of the Institute has had large classes attending his instruction in this department of useful knowledge.

In the last annual report of the Board the crowded condition of every branch of the Institute was clearly presented, and it was stated that "the drawing school alone has room to grow." The drawing school is now occupied to its full capacity, and there is no exception to be made. The necessity for new and enlarged accommodation is more and more apparent.

Under the pressure of this necessity the Board of Managers during the last year authorized to be set on foot a subscription to a building fund of $200,000, and a full committee of the Institute was appointed to obtain subscriptions. The amount subscribed up to the first of the year amounted to over $50,000, contingent upon the whole amount being raised. It will require a steady and persistent effort on the part of the committee, and a generous liberality on the part of the public, to complete this subscription. Meanwhile it is urged upon every member to make known the character and benefits of the Institute, that the membership may be increased.

By order of the Board,

W. P. Tatham, *President.*

The Chairman of the Library Committee presented the following report, which on motion was adopted:

The Library.

The Committee on the Library respectfully report that the number of volumes added to the Library during the past year was—

Volumes.	Bound.	Unbound.	Pamphlets.	Maps.
By purchase,	100	16	93	
Donations received,	243	201	262	4
Exchanges,	162			
Books other than exchanges,	84			
Additions in 1881,	589	217	355	4

The total number of bound volumes in the Library December 31, 1881, was 15,968.

Exchanges.—13 foreign and 7 domestic exchanges for the Journal of the Institute have been added to the list during the past year. The monthly number of exchanges for the Journal is:

Foreign,	103
Domestic,	77
City,	19
——Total,	199

Duplicates.—A list of duplicates has been printed to aid in effecting exchanges. Books to the value of about $69 have been exchanged, and the committee hope that by the aid of the printed list, they will be able to continue the exchange of duplicates for other volumes valuable to the Library.

Serial Publications.—Six valuable serials have been completed during the year. The valuable work of Chief-Engineer Isherwood, "Experimental Researches on the Steam Engine," has been completed by the donation of the first volume by Mr. B. H. Bartol.

The purchase of the photographs of the Suez Canal, with the reports of M. Lesseps and others on its construction, is a valuable addition to our literature on this important work.

The Charts of the United States Coast Survey, secured some two years ago, have been mounted, numbered according to the catalogue, and placed in a suitable case convenient for reference. These charts are now much used by those interested in this department.

The Charts of the Light House Board of the United States, of

which the Library possesses a full set, will, when mounted in the same way, afford a ready reference to them.

The Librarian has prepared a chronological table of American patents published in the Journal of the Institute between the years 1826 and 1859 inclusive. By means of this table, the volume containing the claims of an invention can at once be found, and will be a valuable addition to a set of the Journal. The table is published in the January number of the Journal, 1882.

An alphabetical Index of names of persons to whom patents have been granted in foreign countries is now in progress by the Librarian. When finished, this index will add greatly to the convenience of those having occasion to consult foreign patent records.

We believe that the Library of the Institute is the only Library in the United States wherein can be found the British, Australian, Canadian, French, and United States patent records which are open to the free inspection of the public.

List of Additions to the Library.—By direction of the Committee on Library, an alphabetical list of books added to the Library was printed in the October number of the Journal, 1881, and the second list in the number of January, 1882. It is the intention that this list will be continued quarterly.

Bloomfield Moore Fund.—The annual interest received from the generous donation of $10,000 from the estate of our late highly esteemed fellow-member B. H. Moore, has been expended in the purchase of suitable volumes, which will be appropriately marked, as purchased by the proceeds from this fund. The additions made to the Library from this fund will be a *growing* remembrance of the interest taken by Mr. Moore in the Franklin Institute.

The Library of the Institute now contains one of the most complete collections of works appertaining to science and the arts to be found in America. Its value as a library of reference is attested by the number of persons, both at home and from distant sections, who make use of it for this purpose.

Our present building will soon cease to afford proper accommodations for the Library, and in the event of a disaster from fire in the thickly built up neighborhood, it will imperil a collection which it will be difficult to restore by the money value represented.

CHARLES BULLOCK, *Chairman of Committee on the Library.*

Philadelphia, January, 1882.

The Trustee of the Institute in the Pennsylvania Museum and School of Industrial Art presented the following report, which on motion was adopted:

REPORT OF TRUSTEE OF PENNSYLVANIA MUSEUM, ETC.

Your Trustee has great pleasure in making the following report: That very encouraging progress has been made by the pupils of the Art School under the able instruction of the Principal, Mr. Miller. At an exhibition held last spring, their work was very much admired, and showed the careful way in which they had been taught. The rooms at 1709 Chestnut street are in a very desirable part of the city, and seem admirably adapted for the purpose for which they are used.

With a view of extending the usefulness of the School, the Trustees, at a meeting held last July, decided to establish five free scholarships, to be annually filled by as many deserving pupils from the public schools, and the offer being accepted by the Board of Education, the appointments were at once made, and the pupils have been pursuing their studies ever since. The importance of introducing drawing and the manual arts, as they have been termed, into the schools, is acknowledged by nearly every one, and a very able argument upon this subject and the cultivation of art taste in a great manufacturing community like Philadelphia was delivered last summer by Mr. John S. Clark in this Hall, upon the invitation of the Board of Trade and the Franklin Institute.

If a child is taught nothing else in the schools, he should at least know how to read, to write and to draw correctly, for nothing will be of more importance to him in after life, and if the Museum School by thorough training can fit but a few persons for teaching each year, its usefulness will be acknowledged, and by the increase in its classes it can soon be made self-supporting.

The Museum is now open free to the public every day but Monday, and has attracted many thousands of delighted visitors the past year; indeed, Philadelphians are only beginning to find out what strangers have long known through their guide-books and the reports of friends that we have, owing to the wise liberality of the city and its Park Commissioners, one of the best appointed and most admirably arranged museums to be found in this country, that they may visit at any time, and take as many of their friends to see it as they please, without any charge whatever. The Secretary of the Museum, Mr. Dalton Dorr, has

been untiring in his devotion to this great work, and much credit is due to him.

The additions to the Museum have been quite extensive the past year, consisting of pottery and porcelain in great variety, enamels, glassware ancient and modern, metal work, gold and silver ornaments, ivory and wood carving, all presented by Mrs. Moore, to form the Bloomfield Moore Memorial collection, besides a number of tiles, Roman coins, medals, etc., given by other friends of the Museum.

Your representative has great pleasure in chronicling these gifts, and the bright future that seems about to dawn upon the Museum, convinced that a wise liberality in aiding such institutions cannot help but redound in the highest degree to the welfare and happiness of our people.

ISAAC NORRIS, M.D.

The Chemical Section of the Franklin Institute reported a list of their officers for 1882 and an abstract of their proceedings, and on motion, the report was accepted.

Mr. Grimshaw made a verbal report of progress from the Section of Mechanical Engineering, stating that he would have prepared a written account of its operations if he had known that it was customary or necessary to do so at the annual meeting.

The Committee on a proposed reorganization of the Committee on Science and the Arts reported progress.

The Tellers presented their report of the election, held between the hours of four and eight o'clock, 307 votes having been cast, and the President announced the result of the election, as follows:

President, William P. Tatham.

Vice Presidents, for the full term, J. E. Mitchell; for the unexpired term of Mr. Cartwright (deceased), Frederick Graff.

Secretary, Dr. William H. Wahl.

Treasurer, Frederick Fraley.

Managers—William Sellers, J. Vaughan Merrick, Hector Orr, Cyrus Chambers, Jr., Prof. William D. Marks, William V. McKean, Henry R. Heyl, and Prof. Robert E. Rogers.

Auditor, William B. Cooper.

Trustee in Pennsylvania Museum and School of Industrial Art, Dr. Isaac Norris.

On motion of Mr. Burk, a vote of thanks was tendered to the tellers for their services.

Mr. John Carbutt described the process of taking dry plate photographs, the cheapness of the apparatus, ease of manipulation, etc., and illustrated his remarks by displaying in the lantern many beautiful slides made by amateurs, particularly landscapes and views of animals in the Zoological Garden, taken instantaneously.

Mr. Robert Grimshaw described briefly the process of manufacturing beet root sugar, and said that, if American farmers would accept the experience gained in Europe during the last hundred years in the cultivation of beets for sugar making, they would get as good results as are obtained in Europe and make more money. The only way to compel them to pay attention to the character of the beets they grow is to pay them for the product in proportion to the percentage of crystallizable sugar contained in their beets, and this is being done in Delaware.

In answer to inquiries, Mr. Grimshaw said that the reason American farmers did not get as much from their beet farms as they theoretically should get, was that they did not pay strict attention to the ascertained conditions of perfect success. They seemed to suspect that the manufacturer had some sinister motive when he recommended any particular kind of beet or mode of culture, and by sacrificing the root for top plant, and by failing to gather the crop at the proper time when it contained its highest percentage of crystallizable sugar, they defeated their own purpose and lost money, or failed to make as much as they might have made. Time, however, will correct this evil.

The Secretary's report included a description of the Diamond Steam Engine, which was exhibited by Thomas J. Fales. It is similar to the Baxter engine, but contains some improvements, whereby it is claimed to be made more economical and easier to clean. The cylinder is enclosed in a superheating chamber, into which the exhaust steam passes without coming in contact with the air. It passes thence to a vacuum chamber which holds the coils that feed the boiler, and thence to the hot-air flue, exhausting into the stack, and thereby aiding the draught. The coil for feed water is made of larger pipe than the feed or outlet, thus allowing the water to lie in it longer or until it is heated almost to the boiling point. There are various improvements in mechanical details, whereby it is claimed that the boiler may be more readily cleaned than in the Baxter engine, and the cylinder itself, and, indeed, all parts of the engine, be taken apart and removed without interfering with the boiler if the latter is being used for heating purposes.